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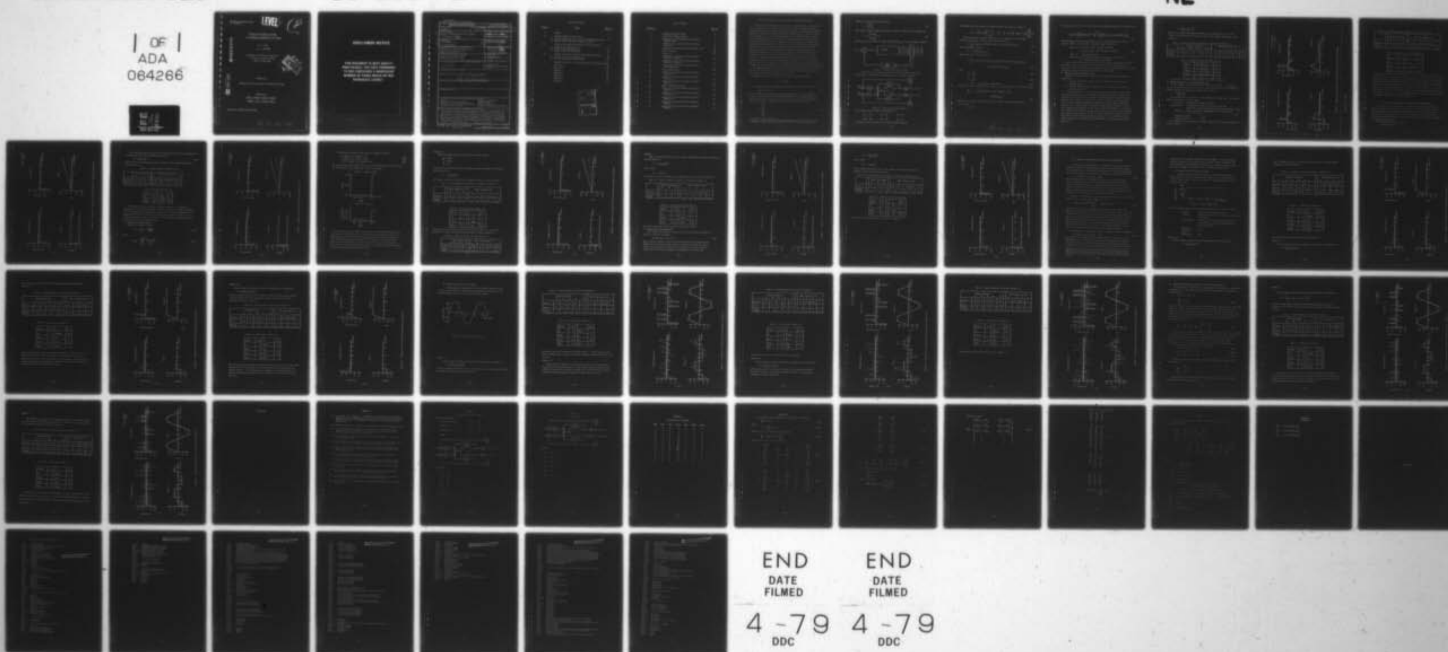

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DESIGN OF FEEDBACK CONTROL AND GEOMETRY PARAMETERS VIA MOFNM

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A modified Newton technique (MOFNM) is presented and applied to the optimization of several design parameters of a remotely piloted vehicle. The technique is programmed to simultaneously optimize the gains of the feedback system and the geometry parameters of the vehicle. A logrithmic transformation is developed, and implemented to guarantee sign definite final design parameters. Several examples are included to illustrate the technique's effectiveness.

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DESIGN OF FEEDBACK CONTROL AND GEOMETRY PARAMETERS VIA MOFNM

A significant breakthrough in geometry design and optimization was achieved in the research effort of 1976-77 [1]. Briefly, a modified Newton technique (Multiple Object-Function Newton Method) was developed and programmed to perform simultaneous optimization of several geometry parameters, e.g., control surface areas, lengths, etc. The potential benefits include improved deflector shapes for existing hull designs, and reduction of design-evaluation-redesign cycle time for completely new crafts. Pursuing this work, two further advances are attempted in the present work. First, gains of the feedback control system and the geometry parameters are collectively considered for optimization. The success of this effort should bring about closer collaboration between the body-shape designer and the control system engineer. Even configurations not considered heretofore could be evaluated rapidly for their true performance potential -- and used when deemed superior by the engineering team. A second improvement considered is in the mathematical formulation of the optimization problem. By use of a logarithmic transformation, the resulting computer solution is sought to be sign definite, and is thereby guaranteed to be physically realizable.

In summary, a methodology for the designer is now available so that he may harness the full potential of body-shape -- including deflection surfaces -- and control gains for maximum performance of the craft.

Examples presented here pertain only to the longitudinal dynamics.

I. THEORY

The linearized state equations of a vehicle* are of the form [3], [5]

$$A \frac{d}{dt} x = Bx + Cu \tag{1}$$

We will assume that the longitudinal and lateral dynamics can be considered decoupled, [4] and thus can be analyzed independently. Concentrating then on longitudinal dynamics, equation (1) can be used to characterize the response of the pitch and depth variables. Specifically, the state vectors become

$$x = \begin{bmatrix} U \\ W \\ \dot{\theta} \\ \theta \\ z \end{bmatrix} \tag{2}$$

---

* The vehicle under consideration is a remotely-piloted vehicle (RPV): It's hydronamic coefficients for longitudinal dynamics are listed in Appendix D.

while the control deflection vector is

$$u = \begin{bmatrix} \delta_b \\ \delta_s \end{bmatrix} \tag{3}$$

The control system configuration shown in Figure 1 results in the feedback law

$$u = D \begin{bmatrix} \theta_{com} \\ Z_{com} \end{bmatrix} + Ex \tag{4}$$

where θ_{com} is the input pitch angle command and Z_{com} is the input depth command.

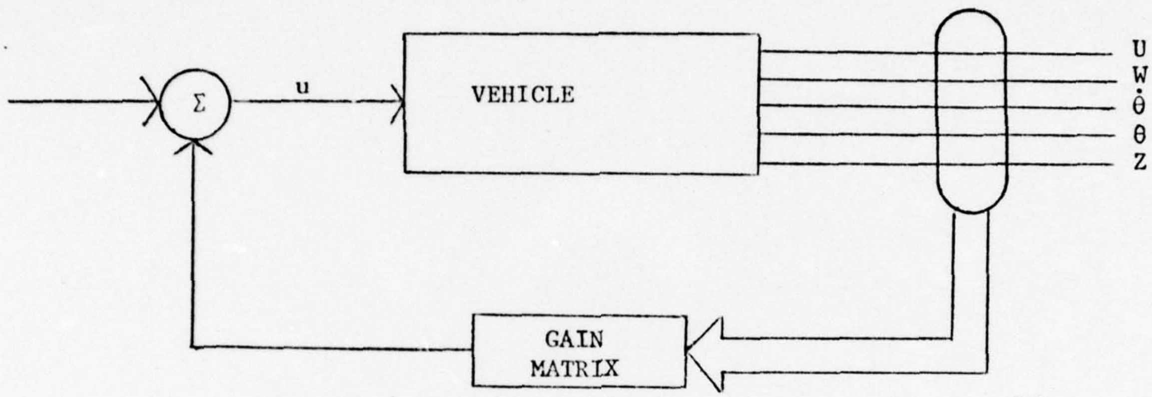


Figure 1. Generalized Control System

In this phase of the study, we will use the stern plane as the only control input. The feedback configuration used is shown in figure 2.

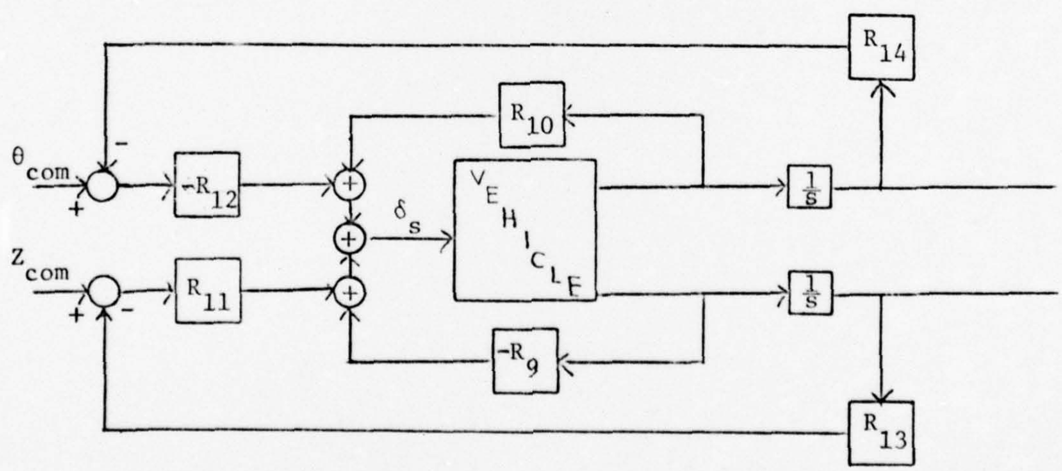


Figure 2. Stern Plane Feedback System

The feedback Gains in figure 2 are defined by

$$R_9 = K^s_{\dot{z}} \qquad R_{11} = K^s_z \qquad R_{13} = K_1$$

$$R_{10} = K^s_{\dot{\theta}} \qquad R_{12} = K^s_\theta \qquad R_{14} = K_2$$

See Apendix A for the definition of other design parameters.

The feedback law which governs the system depicted in figure 2 is given by

$$u = \delta_s = \begin{bmatrix} -K^s_\theta & K^s_z \end{bmatrix} \begin{bmatrix} \theta_{com} \\ Z_{com} \end{bmatrix} + \begin{bmatrix} 0 - K^s_{\dot z} & K_{\dot\theta}\ (K_2K^s_\theta + K^s_{\dot z}U_o) - K_1K^s_z \end{bmatrix} \begin{bmatrix} U \\ W \\ \dot\theta \\ \theta \\ Z \end{bmatrix} \tag{5}$$

Given the design responses $z(k), k=1,2,\ldots,K$, estimates for the optimum feedback and geometry parameters R are found such that

$$J = \sum_{k=1}^{K} [z(k\Delta)-x(k\Delta)]^T Q[z(k\Delta)-x(k\Delta)] + [R_o-R_{\nu+1}]^T P[R_o-R_{\nu+1}] \tag{6}$$

is minimized [6], [8] where

$$\hat{R} = [\ \hat{K}_{\dot z}, \hat{K}_{\dot\theta}, \hat{K}_z, \hat{K}_\theta, \lambda_s\ ]^T \tag{7}$$

Equation (6) can be written [1] as

$$J = \sum_{k=1}^{K} [z(k\Delta)-x_\nu(k\Delta)-H_1H_2(\hat{R}_{\nu+1}-\hat{R}_\nu)]^T Q[z(k\Delta)-x_\nu(k\Delta)-H_1H_2(\hat{R}_{\nu+1}-\hat{R}_\nu)]$$

$$+ [R_o-R_{\nu+1}]^T\ P\ [R_o-R_{\nu+1}] \tag{8}$$

where

$$H_1 = \frac{\partial x}{\partial c^T} \tag{9}$$

$$H_2 = \frac{\partial c}{\partial \hat{R}^T} \tag{10}$$

$$c = f(R) \qquad \text{Hydrodynamic coefficients} \tag{11}$$

Setting the partial derivative with respect to $\hat{R}$ equal to zero, [2], [12] we obtain

$$\frac{\partial J}{\partial R} = 0 = -2\sum_{k=1}^{K} H_2^T H_1^T Q[z(k\Delta)-x_\nu(k\Delta)-H_1H_2(\hat{R}_{\nu+1}-\hat{R}_\nu)]$$

$$-2\bar{R}^T P\bar{R}(\hat{R}_o-\hat{R}_{\nu+1}) \tag{12}$$

where $\bar{R}$ is a diagonal scale matrix for the feedback and geometry parameters $\hat{R}$ such that

$$R = \bar{R}\hat{R}$$

The solution to (12) for the new value of the feedback parameters $\hat{R}$ is given by [1]

$$\hat{R}_{\nu+1}=\hat{R}_{\nu} + \left[H_2^T \sum_{k=1}^{K} H_1^T Q H_1 H_2 + \overline{R}^T \ \overline{PR}\right]^{-1} \left[H_2^T \sum_{k=1}^{K} H_1^T Q(z(k\Delta)-x_{\nu}(k\Delta)+\overline{R}^T \overline{P}\overline{R}(\hat{R}_o-\hat{R}_{\nu})\right] \tag{13}$$

The optimization method oulined above is used in the multiple object function approach, MOFNP, along with the constraint (banded prediction)

$$\begin{aligned} x(k\Delta + \Delta) &= \zeta(k)[z(k\Delta) + \Delta A^{-1} (B\ z(k\Delta) + Cu(k\Delta))] \\ &\quad + (1-\zeta(k))\ [x(k\Delta) + \Delta A^{-1} (B\ x(k\Delta) + Cu(k\Delta))] \end{aligned} \tag{14}$$

$x(0) = x^\circ$ initial conditions

where $\zeta(k)$ is an appropriately chosen sequence of 0's and 1's.

II. FEEDBACK PARAMETER OPTIMIZAION

This section deals with the selection of the feedback parameters of figure 2 that will yield the optimum trajectory with respect to a specified desired trajectory. The stern plane geometry parameter λ_s has been hard-wired to 1.0, thus eliminating geometry parameter optimization for the present.

A. Optimal Design with Doublet Input

Consider the system given in figure 2, excited with the following input combination:

i) 1 degree pitch angle doublet with 12.5 seconds positive and 12.5 seconds negative.

ii) 100 feet depth command doublet with 12.5 seconds positive and 12.5 seconds negative.

It can be shown that, in order to assure system stability, the following conditions must be met: 1) The feedback gains R_{13} and R_{14} (see figure A1 in Appendix A) must provide unity feedback in order to generate the actual pitch and depth error signals. 2) Since the numerical value of the depth command is two to three orders of magnitude larger than the pitch angle command, K_θ should be two to three orders of magnitude larger than K_z in order to assure that comparable contributions to the control input are produced. 3) The depth rate feedback gain $K_{\dot{z}}$ must be chosen to be small to prevent excessive overshoot and ringing in the response. 4) The pitch rate feedback gain $K_{\dot{\theta}}$ was chosen to be about two orders of magnitude greater than $K_{\dot{z}}$ to control the pitch rate. These conditions were applied to the optimization of the stern plane feedback parameters,

$$R = \left[\hat{K}_{\dot{z}}, \hat{K}_{\dot{\theta}}, \hat{K}_{z}, \hat{K}_{\theta}\right]^T$$

where λ_s, the stern plane geometry parameter, was hard-wired to 1.0. The results of the experiment are given in table 1; the program settings are given in Table 2.

Table 1. Feedback Parameters and Errors, Doublet Input Response

	Feedback Parameters				RMS % Difference (%)				
	$K_{\dot{z}}(R_9)$	$K_{\dot{\theta}}(R_{10})$	$K_z(R_{11})$	$K_\theta(R_{12})$	U	W	$\dot{\theta}$	θ	Z
Baseline	0.02	1.80	0.05	9.50	75.8	67.2	67.9	77.1	159.7
Optimal Design	0.01	2.00	0.01	9.99	.0023	.002	.0028	.0014	.0015

Table 2. Program Data, Doublet Input Response

NPT	200		NA	5
NS	5		MC	2
IAOPT	1		INTR	5
DELTA	0.5		FACTOR	0

The desired and final design responses are given in figure 3.

B. Optimal Design with Pitch Command Step Input

In this example, the control system of figure 2 is configured as follows: i) K_z and $K_{\dot{z}}$ are hard-wired to zero ii) λ_s is hard-wired to 1.0. The remaining design parameters are chosen for optimization. That is,

$$\hat{R} = \left[\hat{K}_{\dot{\theta}}, \hat{K}_{\theta}\right]^T$$

The command input is taken to be a -25° degree pitch angle step. The desired system responses are taken as follows

Pitch (θ) -25 degree pitch angle (θ) step

Pitch Rate ($\dot{\theta}$) -50 degree/sec. pitch rate ($\dot{\theta}$) pulse, one half second wide to allow leading edge of pitch angle step to occur.

Depth (Z) Depth response (Z) corresponding to the relationship

$$Z = W - U_o\,\theta \qquad (15)$$

Forward Velocity u - zero

Plunge Velocity w - zero

The results of this experiment are given in Table 3; the program settings are listed in Table 4.

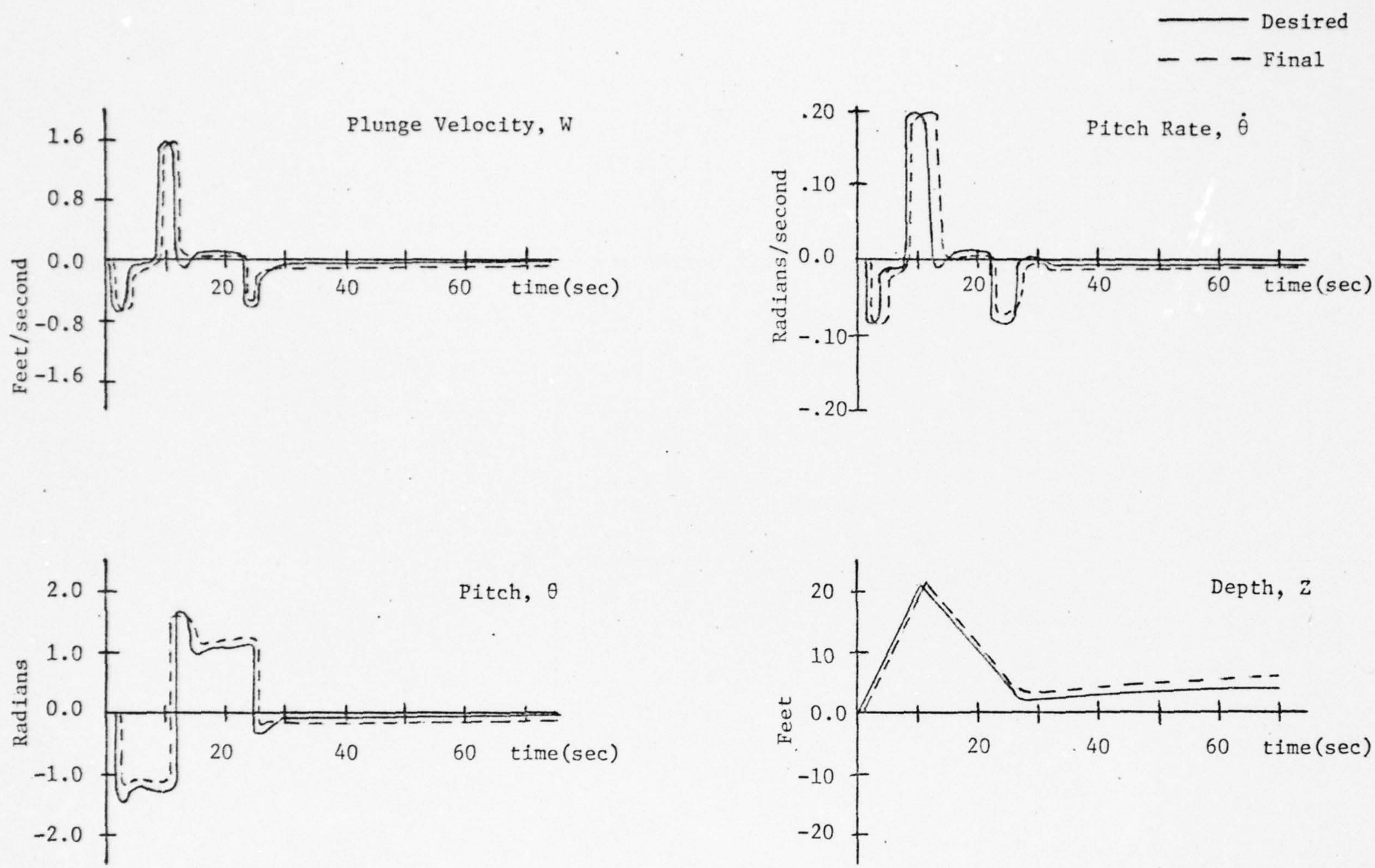


Figure 3. Comparison of Desired and Final Responses, Feedback Optimization

Table 3. Design Parameters and Errors, Pitch Step Response

	Design Parameters $K_{\dot{\theta}}$	K_{θ}	λ_s^2	RMS % Difference (%) U	W	$\dot{\theta}$	θ	Z
Baseline	0.0	0.0	1.0	0.0	0.0	87.3	615.5	-
Optimum Design	5.07	13.71	1.0	100.0	100.0	92.1	5.42	106.9

Table 4. Program Data, Pitch Step Response

Factor	0	P	0
NS	5	MC	2
IAOPT	1	INTR	5
DELTA	0.5	NPT	200

The desired and final design responses are shown in figure 4. Note that the desired responses for $\dot{\theta}$ and Z are chosen so as to be compatible with a -25 degree pitch angle step. Observation of the RMS% Differences reveals that four of the trajectories differ significantly from the desired trajectories. It should be noted, however, that although desired trajectories were specified for all five states, only the pitch angle (θ) was actually optimized. This is true because all entries of the Q matrix except Q(4,4), which corresponds to the pitch angle, were hard-wired to zero. The only significant error, therefore, is that of the pitch angle response, which is relatively small.

III. GEOMETRY AND FEEDBACK OPTIMIZATION (PITCH MANEUVERS)

In this phase of investigation, a combination of feedback and geometry parameter optimization using the stern plane model of figure 2 was attempted. The experimentation centers around the development of an effective method of calculating the weighting matrix P. All case examples use the pitch angle step input and desired response specifications given in the previous experiment.

A. System Design with P=0

Let the matrix P be set to zero, i.e.

P(i,i)=0 for all i

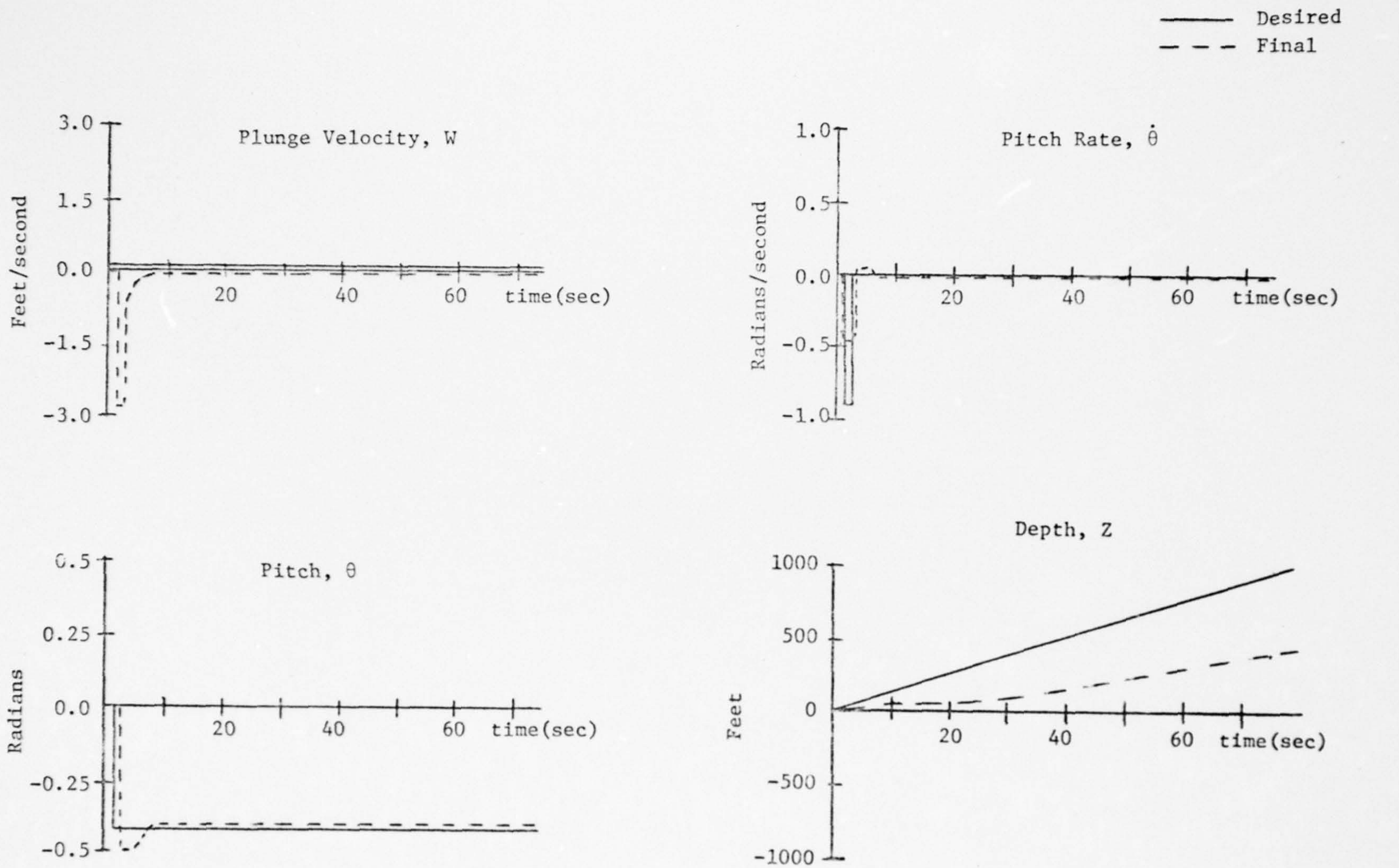


Figure 4. Comparison of Desired and Final Responses, Feedback Optimization

The feedback parameters K_z and $K_{\dot{z}}$ are hard-wired to zero so the optimization procedure dealt with the following parameter set

$$\hat{R} = [\hat{K}_{\dot{\theta}}, \hat{K}_{\theta}, \hat{\lambda}_s^2]^T \tag{16}$$

The results of the experiment are given in Table 5; the program settings are listed in Table 6.

Table 5. Design Parameters and Errors, P=0

	Design Parameters			RMS % Difference (%)				
	K_θ	$K_{\dot{\theta}}$	λ_s^2	U	W	θ	$\dot{\theta}$	Z
Baseline	0.0	0.0	1.0	0.0	0.0	87.3	615.5	-----
Optimal Design	5327	3468	.009	100	100	117.2	3.5	106.5

Table 6. Program Settings, P=0

FACTOR	0		P	0
NS	5		MC	2
IAOPT	1		INTR	5
DELTA	0.5		NPT	200

The desired and final responses are given in figure 5. Although an improvement in the pitch angle error has been achieved, the feedback parameter values are quite large in comparison to those of the previous experiment. The program MOF-NP has the capability to penalize large departures from a set of a priori parameters. This is achieved by calculating the weighting matrix P of equation (6) using the adaptive method.

B. Design Using Adaptive Method

Consider the matrix P given by

$$P(i,i) = \frac{1.0}{\hat{c}_{io}^2} \cdot \frac{\text{FACTOR}}{\text{PARER}} \tag{17}$$

where

$$\text{PARER} = \sum_{k=1}^{\text{NPABC}} \frac{(\hat{c}_{ko} - \hat{c}_k)^2}{\hat{c}_{ko}^2} \tag{18}$$

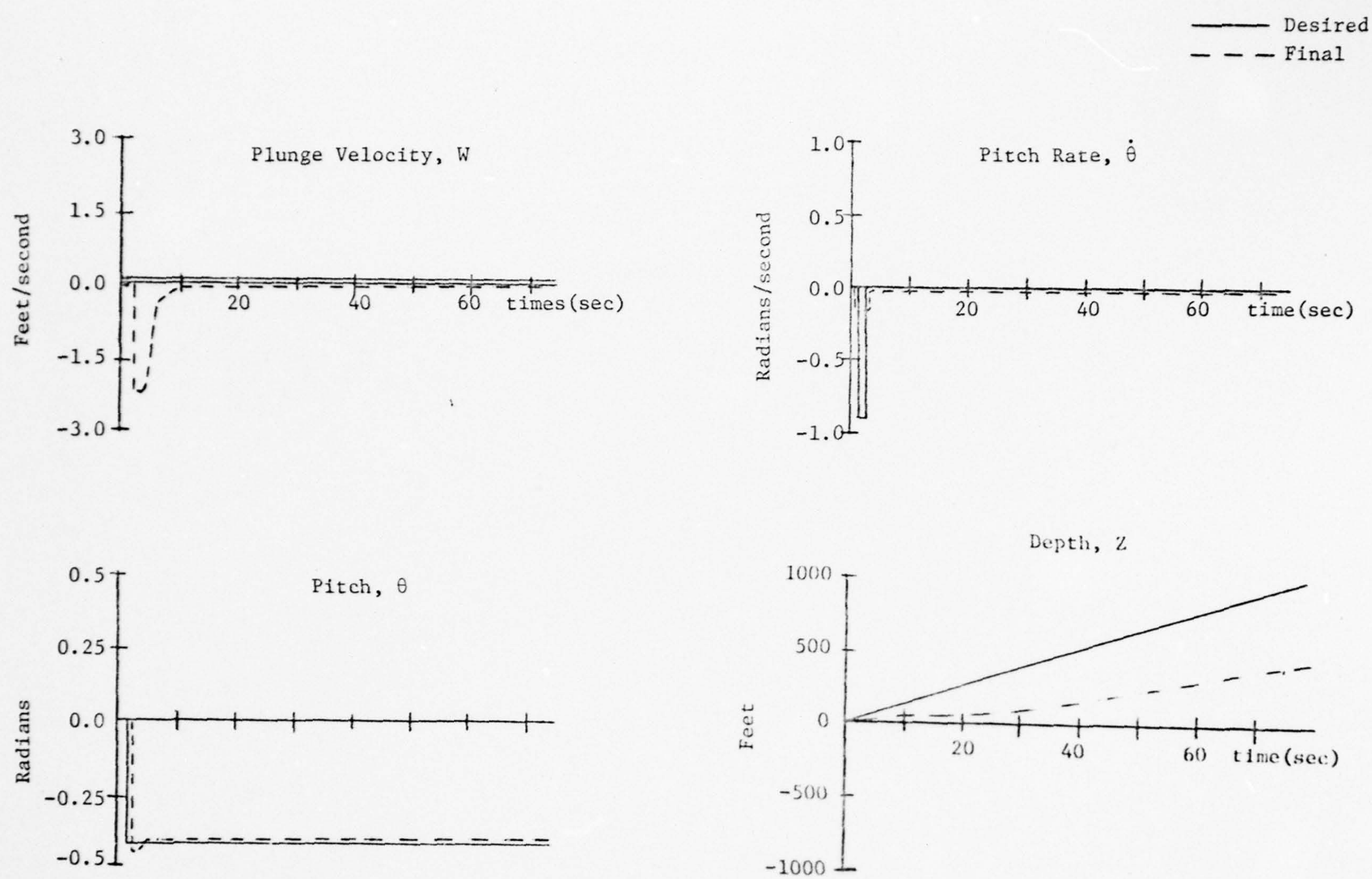


Figure 5. Comparison of Desired and Final Responses, P=0

The following constraints were applied to PARER and FACTOR,

$$\text{if PARER} \leq 0.2,\ \text{FACTOR} = 0.0 \tag{19a}$$

$$\text{if PARER} \geq 1.0,\ \text{FACTOR} = \text{PARER} \tag{19b}$$

$$\text{if } 0.2 < \text{PARER} < 1.0,\ \text{FACTOR} = 0.05 \tag{19c}$$

The relationship between PARER and factor is shown graphically in figure 6, and the relationship between PARER and P is given in figure 7.

Figure 6. PARER vs FACTOR

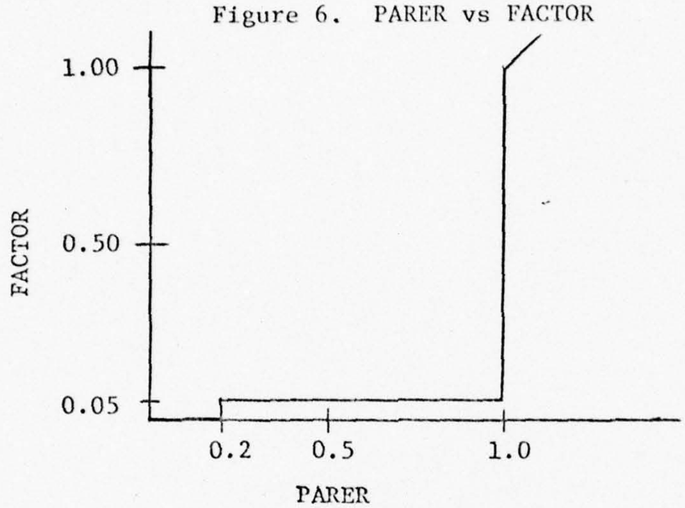
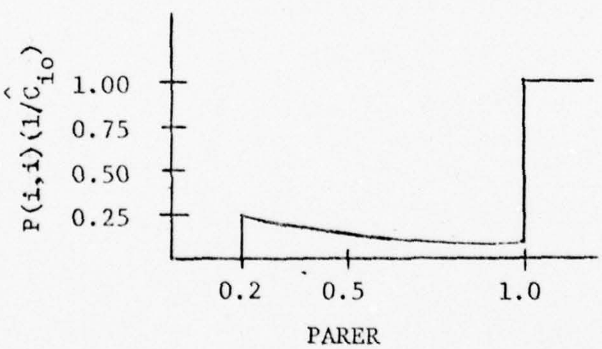


Figure 7. PARER vs P



It is seen, from (18), that PARER is a measure of the normalized deviations of $\hat{c}_k$ from the a priori values, $\hat{c}_{ko}$. Thus, without the constraints (19), as $\hat{c}_k$ approached $\hat{c}_{ko}$, P would become large and only small variations of parameters from a priori values would be allowed. The constraints of (19), however, frees the optimization process from penalties for departures from a priori values when the parameter estimates are close to a priori values, thus allowing a greater degree of optimization flexibility.

Example B-1

Using the design values of section 2-B as apriori values,

$$K_{\dot\theta} = 5.068$$
$$K_{\theta} = 13.713$$
$$\lambda_s^2 = 1.0$$

with K_z and $K_{\dot z}$ hard-wired to zero, feedback and geometry parameter optimization was performed for

$$\hat{R} = [\hat{K}_{\dot\theta}, \hat{K}_{\theta}, \hat{\lambda}_s^2]^T$$

The results are given in Table 7; the program setting are listed in Table 8.

Table 7. Design Parameters and Errors, P adaptive, Example B-1

	Design Parameters K_θ	$K_{\dot\theta}$	λ_s^2	RMS % Difference (%) U	W	$\dot\theta$	θ	Z
Baseline	0.0	0.0	1.0	0.0	0.0	87.3	615.5	-----
Optimal Design	5.11	14.41	1.07	100	100	83.9	5.11	106.8

Table 8. Program Setting, Example B-1

FACTOR	0.05	P	ADP
NS	5	MC	2
IAOPT	1	INTR	5
DELTA	0.5	NPT	200

The desired and final design responses are given in figure 8. A comparison between the a priori design and the final design is given in Table 9.

Table 9. A priori vs final design, Example B-1

	Design Parameters K_θ	$K_{\dot\theta}$	λ_s^2	RMS % Difference (%) U	W	$\dot\theta$	θ	Z
A Priori	13.71	5.07	1.0	100	100	92.1	5.43	106.9
Final Design	14.41	5.11	1.07	100	100	83.9	5.11	106.8

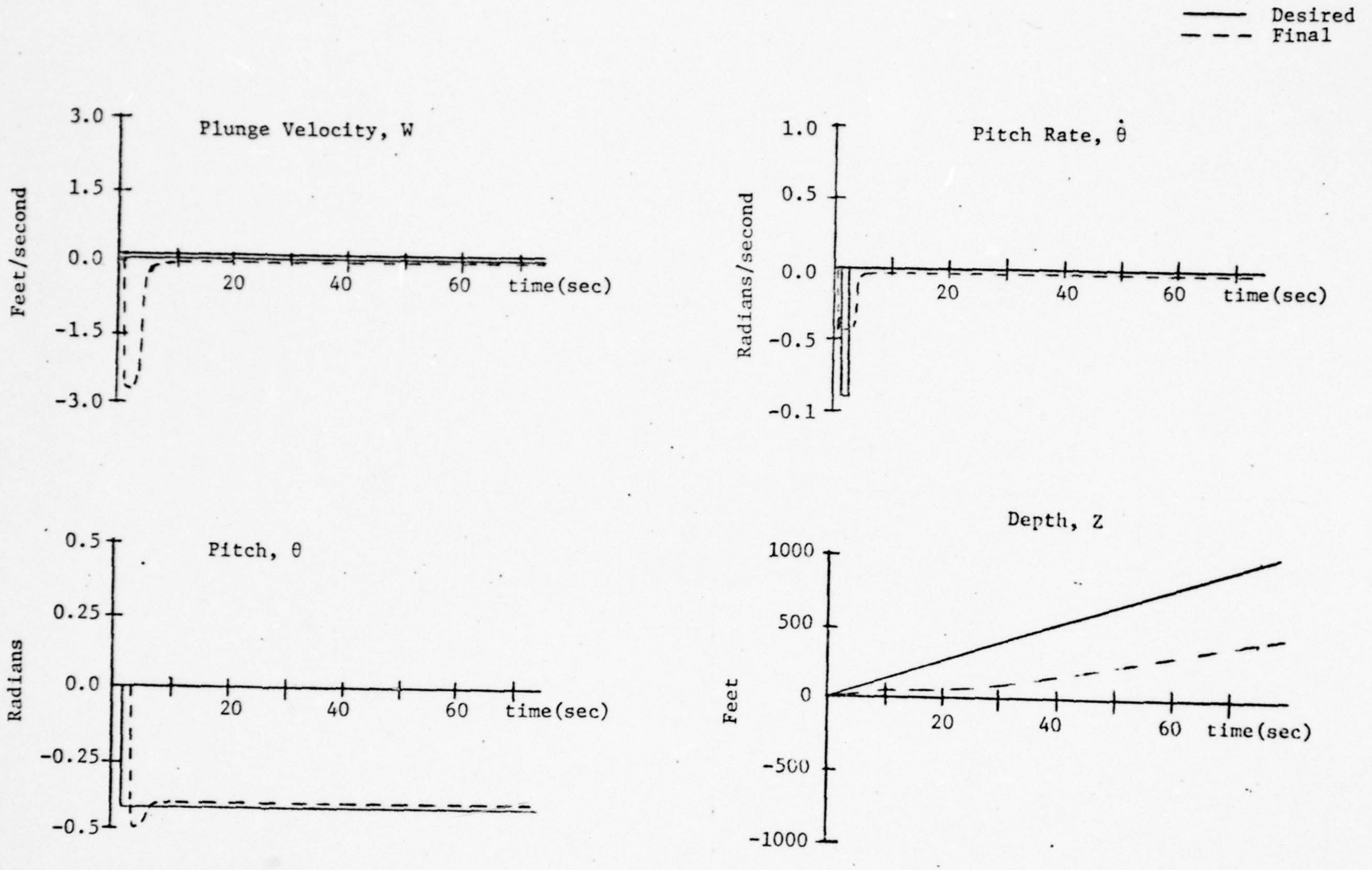


Figure 8. Comparison of Desired and Final Responses, Example B-1 Adaptive

Example 2

With K_z and $K_{\dot{z}}$ hard-wired to zero, feedback and geometry parameter optimization was performed for

$$\hat{R} = [\hat{K}_{\dot{\theta}}, \hat{K}_{\theta}, \hat{\lambda}_s^2]^T$$

with a priori

$$\hat{R}_o = [0,0,1]^T$$

The results are given in Table 10; the program settings are listed in Table 11.

Table 10. Design Parameters and Errors, P adaptive, Example B-2

	Design Parameters K_θ	$K_{\dot{\theta}}$	λ_s^2	RMS % Difference (%) U	W	$\dot{\theta}$	θ	Z
Baseline	0	0	1.0	0.0	0.0	87.3	615.5	-----
Optimal Design	0.87	-0.11	1.20	100	100	345.9	16.2	125.4

Table 11. Program Settings, Example B-2

FACTOR	0.05	P	ADP
NS	5	MC	2
IAOPT	1	INTR	5
DELTA	0.5	NPT	200

The desired and final design responses are given in figure 9.

C. Design using JOPT2 adjustment

In an attempt to improve the trajectory fit of the previous example, the following condition was imposed,

$$\text{if JOPT2} \leq 8, \text{ factor} = 0.0 \tag{20}$$

This condition completely frees the optimzation process from penalties for departures from a priori values during the first four optimization passes (first eight iterations). Using (20) in conjunction with the P adaptive method, (17), (18) and (19), parameter optimization was performed for

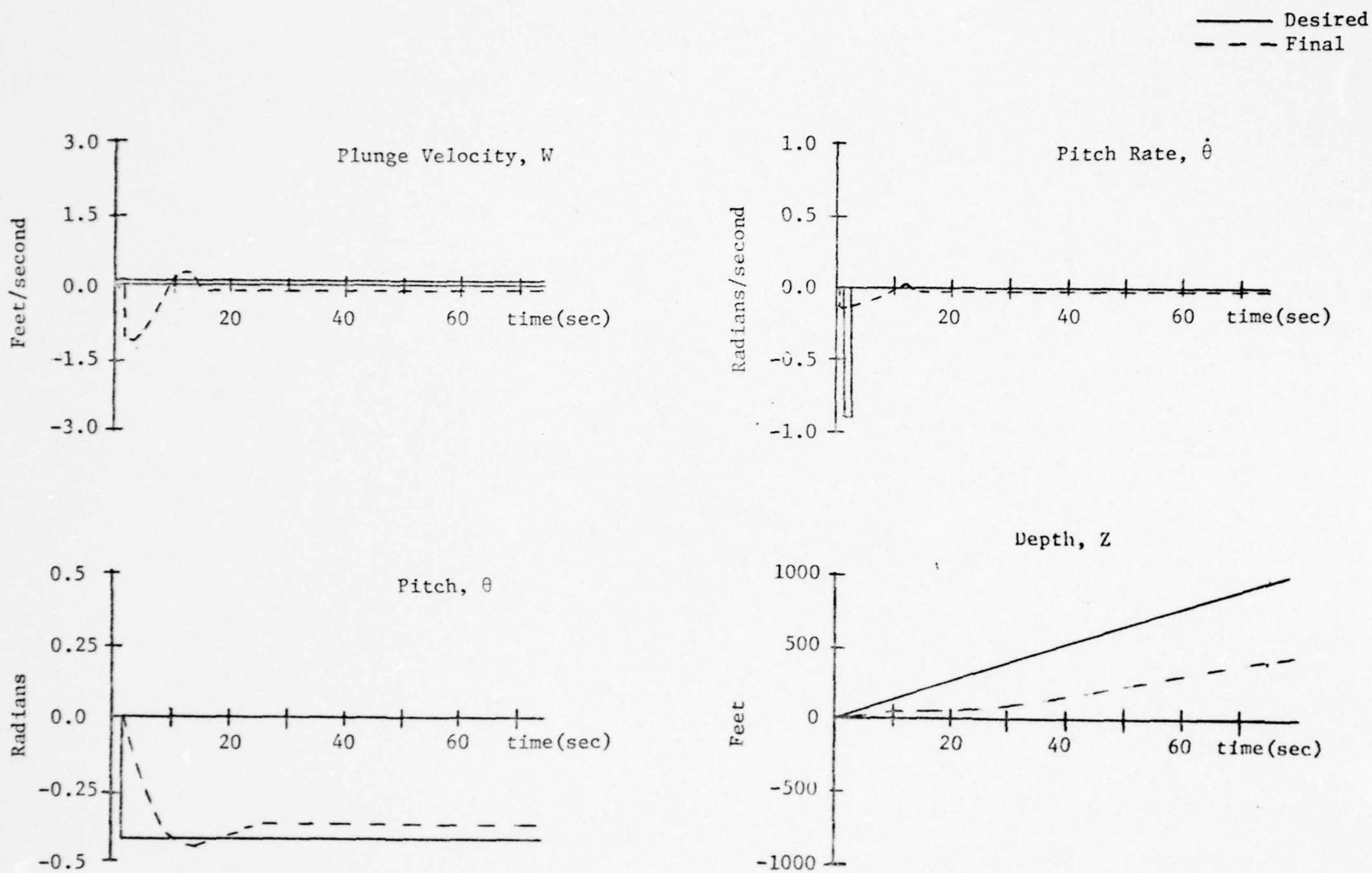


Figure 9. Comparison of Desired and Final Responses, Example B-2 Adaptive

$$\hat{R} = [\hat{K}_{\dot{\theta}}, \hat{K}_{\theta}, \hat{\lambda}_s^2]^T$$

with a priori

$$\hat{R}_o = [0,0,1]^T$$

and K_z and $K_{\dot{z}}$ hard-wired to zero. The results are given in Table 12; the program settings are listed in Table 13.

Table 12. Design Parameters and Errors, P adaptive, JOPT2 adjustment

	Design Parameters K_θ	$K_{\dot{\theta}}$	λ_s^2	RMS % Difference (%) U	W	$\dot{\theta}$	θ	Z
Baseline	0.0	0.0	1.0	0.0	0.0	87.3	615.5	----
Optimum Design	9.93	.115	7.03	100	100	103.9	1.19	105.8

Table 13. Program Data, JOPT2 adjustment

FACTOR	0.05	P	ADP
NS	5	MC	2
IAOPT	1	INTR	5
DELTA	0.5	NPT	200

The desired and final responses are given in figure 10.

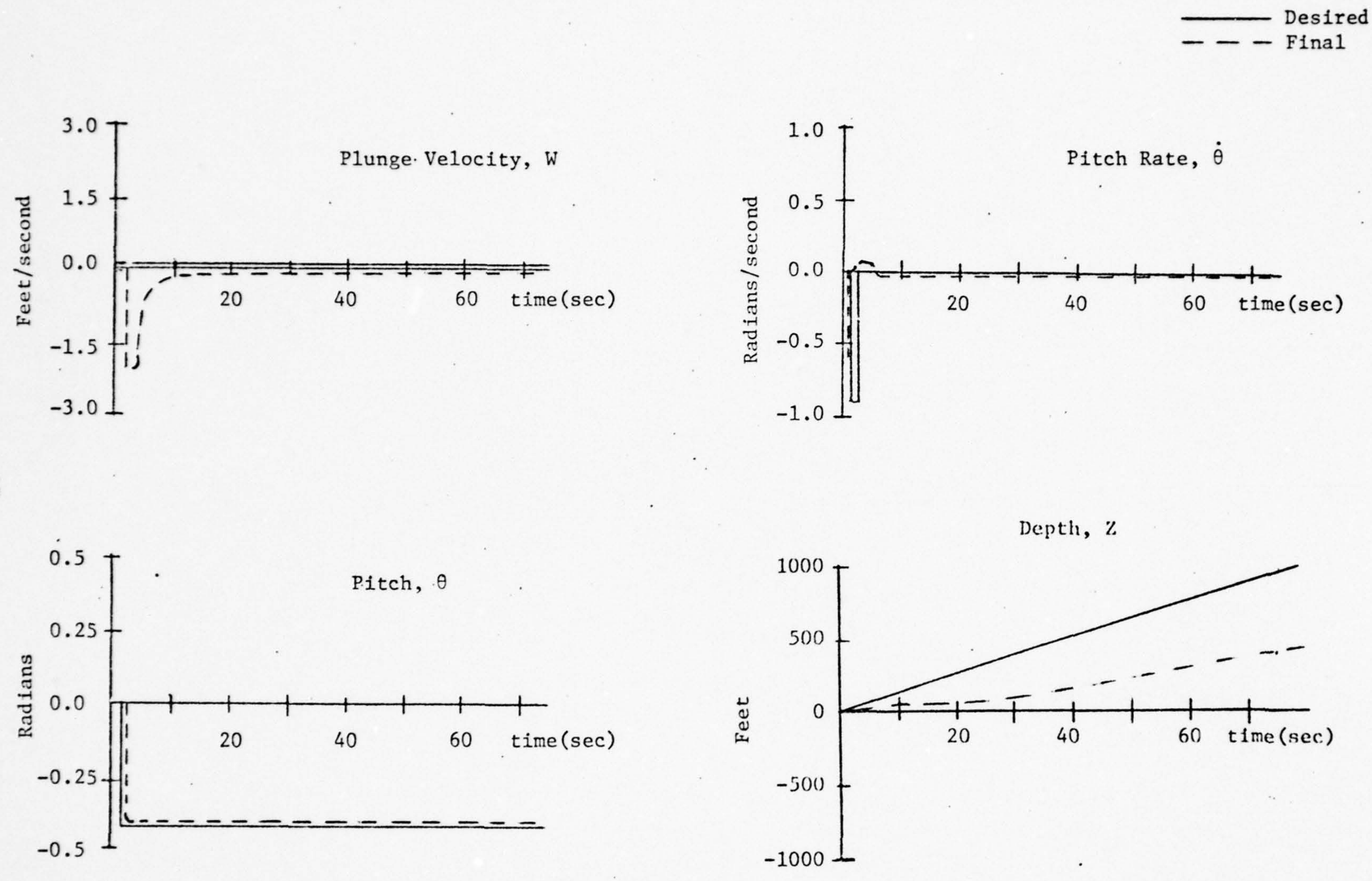


Figure 10. Comparison of Desired and Final Responses, JOPT2 Adjustment

IV. FEEDBACK AND GEOMETRY OPTIMIZATION (DEPTH MANEUVERS)

In this phase of investigation, a refinement of the Adaptive Method given in section 3.B is developed and applied to parameter optimization for depth maneuvers. The original implementation of the Adaptive Method, given in (19), has the upper bound for parameter variations controlled by

$$\text{if PARER} \geq 1.0, \text{ FACTOR} = \text{PARER} \qquad (19b)$$

where PARER is given by (18) and P is given by (17). With (19b) in effect, PARER greater than 1.0 will result in large penalties in the parameter optimization scheme. This method is effective if the priori values are good estimates of the actual optimal system parameters. If, however, the priori values are not close to the optimal parameter values, (19b) may severely handicap the optimization procedure.

Consider, for example, the case where some parameters have priori values equal to zero. In this case, PARER is given by

$$\text{PARER} = \Sigma \hat{r}_k^2 + \Sigma \frac{(\hat{r}_{ko} - \hat{r}_k)^2}{\hat{r}_{ko}^2} = \text{PARER1} + \text{PARER2} \qquad (21)$$

where PARER1 is determined by parameters with zero priori values and PARER2 is determined by parameters with non-zero priori values. If, at any time during the optimization procedure, any zero priori parameter takes a value greater than 1.0 (absolute value), (19b) sets FACTOR equal to PARER and the resulting penalty is large. In fact, examination of (17) shows that the weighting matrix P is no longer a function of either FACTOR or PARER in this case, but is set to 1.0. This scheme, therefore, will not allow a zero-priori parameter to exceed unity (absolute value).

An improvement in the Adaptive Method is made when (19b) is changed to

$$\text{If PARER} \geq \text{MAXER}, \text{ FACTOR} = 1.0 \qquad (22)$$

where MAXER is a variable, usually chosen* between 1 and 300. In (22), when PARER exceeds the designated upper bound, the penalty for departure from priori values is increased significantly (factor typically is increased by one to two orders of magnitude), while the weighting P remains

---

*MAXER should be chosen according to the anticipated variation of parameters from the specified priori values. For example, a MAXER of 400 would allow a zero-priori parameter to assume values up to 20 (absolute value).

a function of the parameter and priori values (PARER). In the case where geometry parameters are specified with zero priori values, a MAXER of 200 has been found to yield reasonable results (for a comparison of optimization efficiency of MAXER = 1 vs MAXER = 200, see example 4.B2 and 4.B3).

A) Optimal Design with Ramp-Step Input

Consider the system given in figure 2, excited with the Depth Command given in figure 11 and specified as follows: (1) Input ramp with 2.5 feet/second slope for time $\leq$ 10 seconds, (2) Input constant at 25 feet for time > 10 seconds.

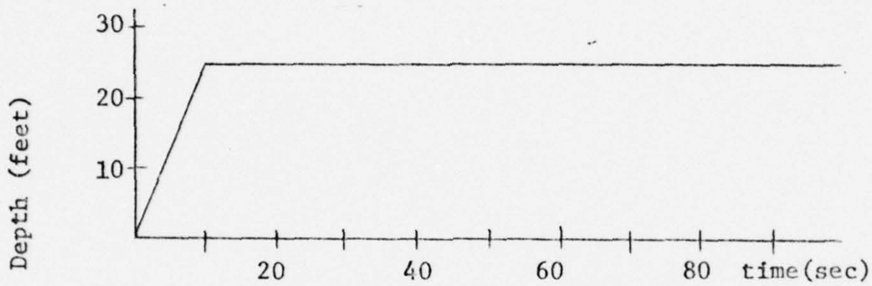


Figure 11. Depth Command (Zcom)

In the following examples the desired system responses are taken as follows:

Depth(Z)	same as Depth Command (figure 11)
Pitch(θ)	$-25/U_o$ pitch angle pulse for time < 10 seconds 0 elsewhere
Pitch Rate($\dot{\theta}$)	(+) $5.0/U_o$ pitch rate ($\dot{\theta}$) pulse, one half second wide at leading (trailing) edge of pitch angle pulse.
Forward Velocity(u)	zero
Plunge Velocity(w)	zero

Example A-1

In this example, feedback parameter optimization is performed for

$$\hat{R} = [\hat{K}_{\dot{Z}}, \hat{K}_{\dot{\theta}}, \hat{K}_Z, \hat{K}_\theta]^T$$

with λ_s^2 hardwired to 1.0. The results are given in Table 14; the program settings are listed in Table 15.

Table 14. Feedback Parameters and Errors, Example A-1

	Feedback Parameters K_θ	K_Z	$K_{\dot\theta}$	$K_{\dot Z}$		RMS % Difference (%) W	$\dot\theta$	θ	Z
Baseline	0.0	0.0	0.0	0.0	-	0.0	48.38	74.56	----
Priori	0.0	0.0	0.0	0.0	-	-	-	-	-
Optimal Design	1.27	.029	1.02	.079	-	100	141.1	57.67	4.79

Table 15. Program Data, Example A-1

FACTOR	0.01	P	ADP1
NS	5	MAXER	200
IAOPT	1	MC	2
DELTA	0.5	NPT	200
MAXR	16	UNKR	4

The desired and final responses are given in figure 12.

Example A-2

In this example, feedback and geometry optimization is performed for

$$\hat{R} = [\hat{K}_{\dot Z}, \hat{K}_{\dot\theta}, \hat{K}_Z, \hat{K}_\theta, \hat{\lambda}_s^2]^T$$

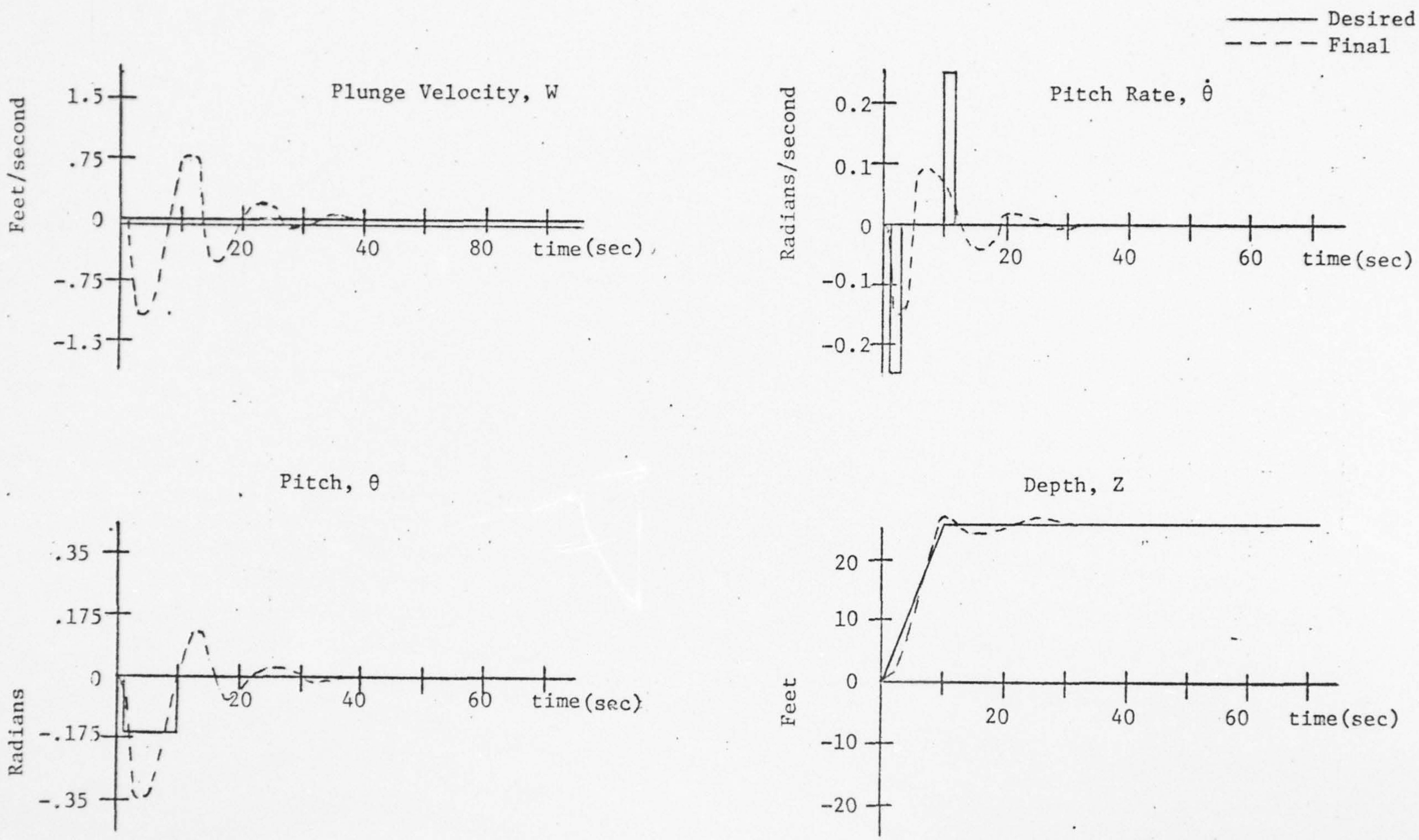


Figure 12. Comparison of Desired and Final Responses, Example A-1

The results are given in Table 16; the program settings are listed in Table 17.

Table 16. Design Parameters and Errors, Example A-2

	Design Parameters K_θ	K_Z	$K_{\dot\theta}$	$K_{\dot Z}$	λ_s^2	RMS % Difference (%) W	$\dot\theta$	θ	Z
Baseline	0.0	0.0	0.0	0.0	1.0	0.0	48.38	74.56	----
Priori	0.0	0.0	0.0	0.0	1.0	-	-	-	-
Optimal Design	.093	.027	.214	.131	1.76	100	137.9	57.8	3.94

Table 17. Program Data, Example A-2

FACTOR	0.01	P	ADP1
NS	5	MAXER	200
IAOPT	1	MC	2
DELTA	0.5	NPT	200
MAXR	16	UNKR	5

The desired and final responses are given in figure 13. Notice that the optimal values obtained for K_Z, $K_{\dot Z}$ and $K_{\dot\theta}$ look reasonable, but K_θ seems inappropriately small. This is, however, a depth manuever, and as such the parameters obtained are acceptable, although it is possible that the pitch response with this set of parameters is poor.

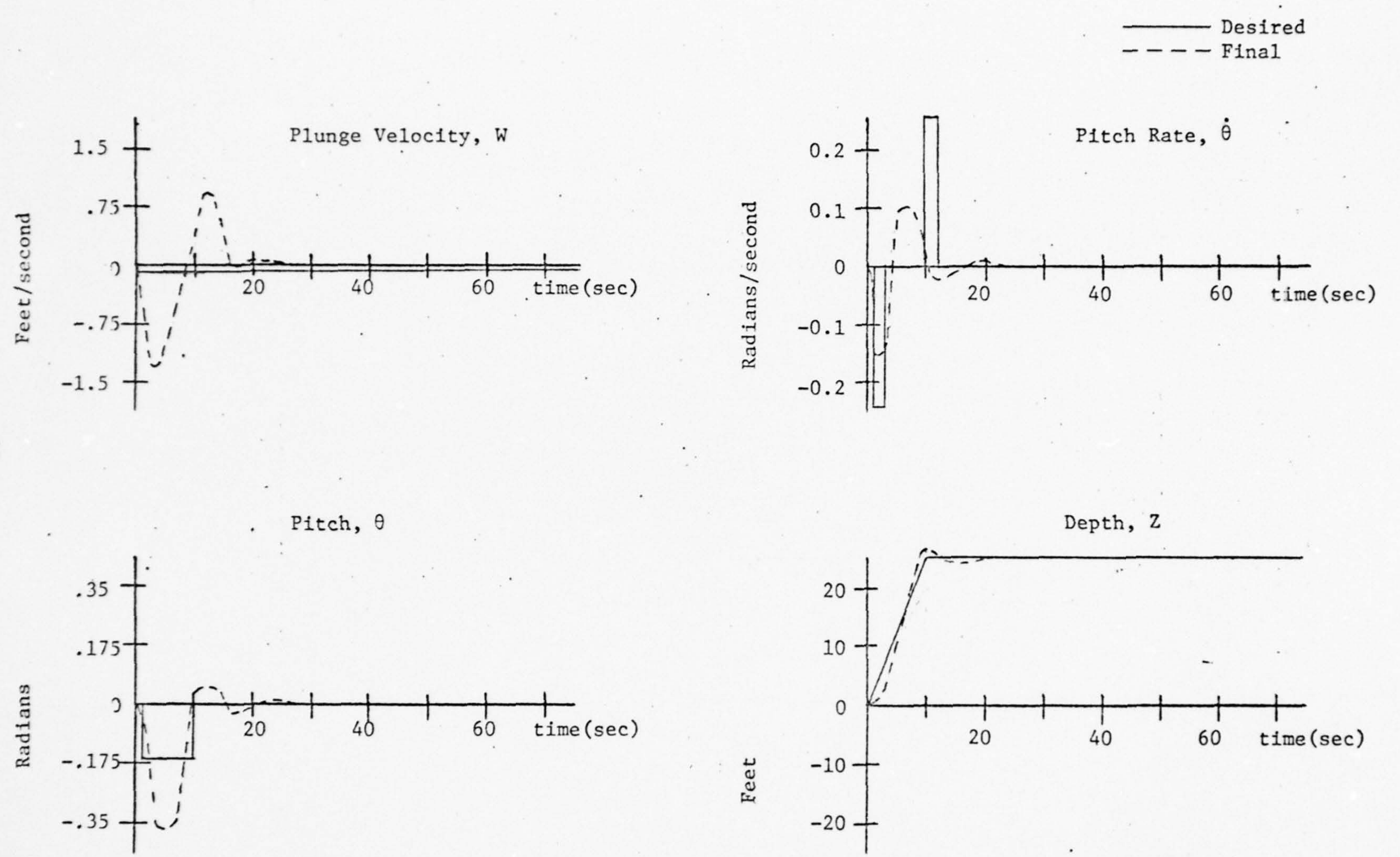


Figure 13. Comparison of Desired and Finals Responses, Example A-2

Example A-3

In this example, feedback and geometry optimization is performed for

$$\hat{R} = [\hat{K}_{\dot{Z}}, \hat{K}_{\dot{\theta}}, \hat{K}_Z, \hat{K}_\theta, \hat{\lambda}_s^2]^T$$

where the priori values for the parameters are based on the results obtained in section 3C(pg. 16). The results are given in Table 18; the program settings are listed in Table 19.

Table 18. Design Parameters and Errors, Example A-3

	Design Parameters K_θ	K_Z	$K_{\dot{\theta}}$	$K_{\dot{Z}}$	λ_s^2	RMS % Difference (%) W	$\dot{\theta}$	θ	Z
Baseline	9.93	0.0	0.115	0.5	7.03	0.0	48.38	74.56	----
Priori	9.93	0.0	0.115	0.5	7.03	-	-	-	-
Optimal Design	9.07	.213	.115	.442	6.57	100.0	93.2	61.8	2.83

Table 19. Program Data, Example A-3

FACTOR	0.01	P	ADP1
NS	5	MAXER	200
IAOPT	1	MC	2
DELTA	0.5	NPT	200
MAXR	16	UNKR	5

The desired and final responses are given in figure 14. Notice that the values obtained here for K_θ and λ_s^2 are quite different from those obtained in the previous example. These parameters are comparable to those obtained in the pitch optimization, and therefore may be acceptable for both depth and pitch maneuvers.

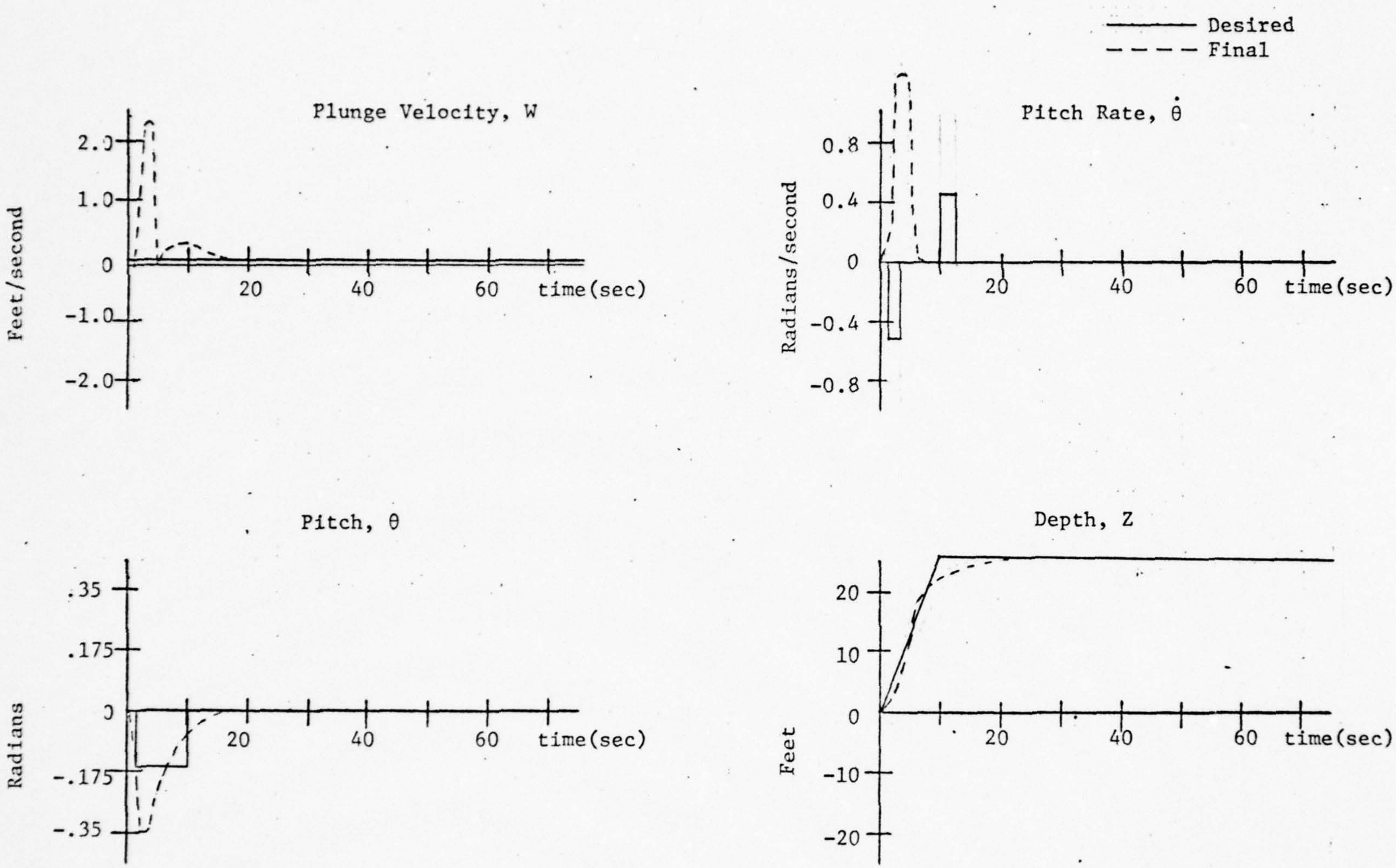


Figure 14. Comparison of Desired and Final Responses, Example A-3

B) Optimal Design with Triplet Input

In this phase of investigation, the stern plane feedback system is excited with the depth command (Z_{com}) input given in figure 16. The desired depth response is the same as the depth command.

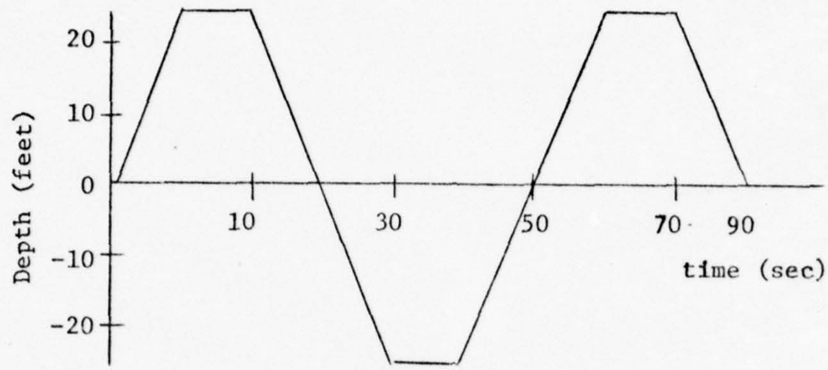


Figure 16. Depth Command Input (Z_{com})

Example B-1

In this example, feedback and geometry optimization is performed for

$$\hat{R} = [\hat{K}_{\dot{Z}}, \hat{K}_{\dot{\theta}}, \hat{K}_{Z}, \hat{K}_{\theta}, \hat{\lambda}_s^2]^T$$

with the feedback priori values set to zero. The results are given in Table 20; the program settings are listed in Table 21.

Table 20. Design Parameters and Errors, Example B-1

	Design Parameters K_θ	K_Z	$K_{\dot\theta}$	$K_{\dot Z}$	λ_s^2	RMS % Difference (%) W	$\dot\theta$	θ	Z
Baseline	0.0	0.0	0.0	0.0	1.0	0.0	96.7	186.6	----
Priori	0.0	0.0	0.0	0.0	1.0	-	-	-	-
Optimal Design	.371	.074	.296	.106	2.19	100	152.9	59.5	23.63

Table 21. Program Data, Example B-1

FACTOR	0.01	P	ADP1
NS	5	MAXER	1
IAOPT	1	MC	2
DELTA	0.5	NPT	200
MAXR	16	UNKR	5

The desired and final response are given in figure 17. Notice that, in this example, MAXER equals 1 and all of the final feedback parameter values are less than 1.0.

Example B-2

This example is identical to the previous example, with the exception that MAXER is set at 200 instead of 1. The results, given in Table 22, show a definite improvement in the depth response error. Notice that some of the feedback parameters (K_θ,$K_{\dot\theta}$) have absolute values greater than 1.0.

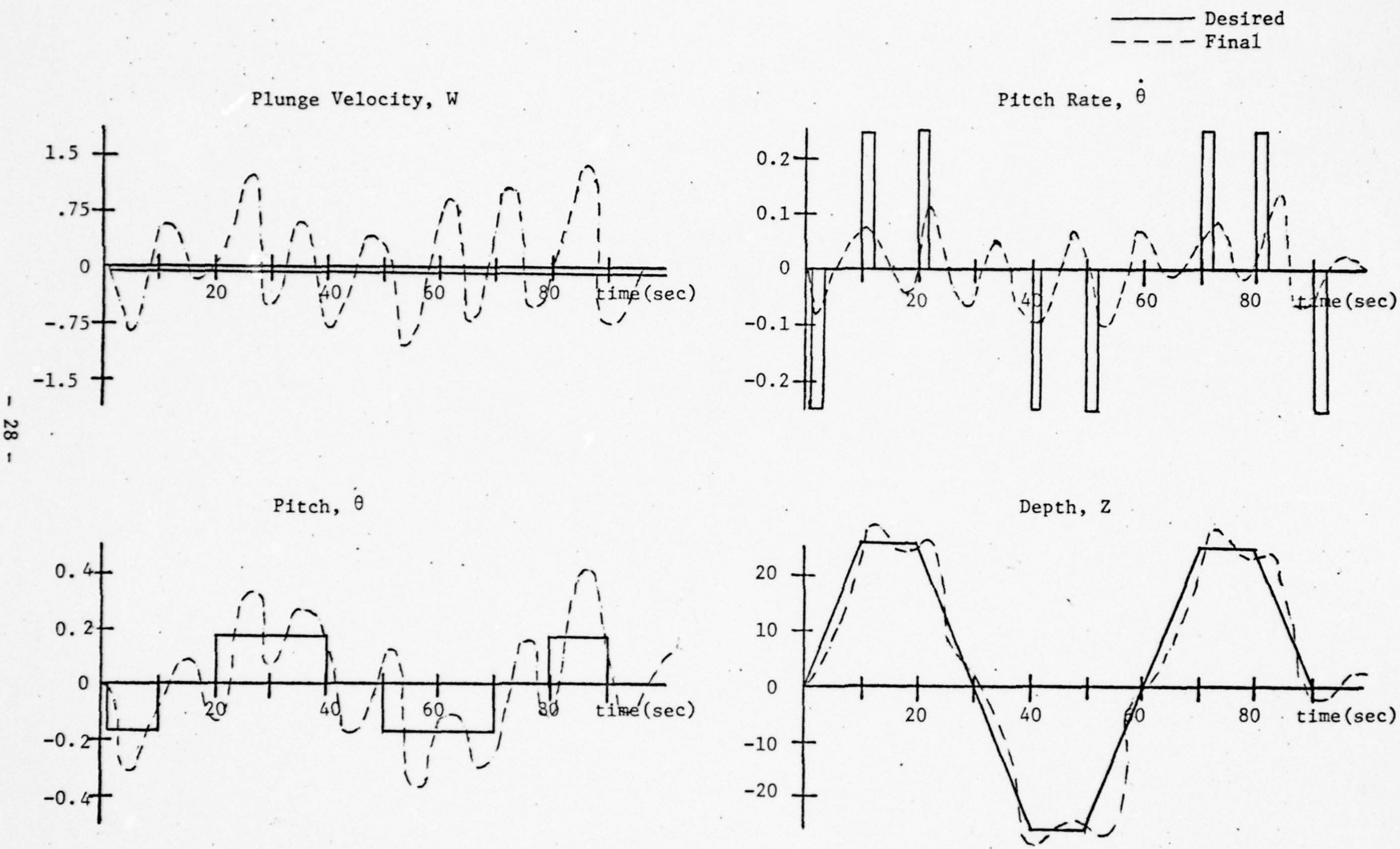


Figure 17. Comparison of Desired and Final Responses, Example B-1

Table 22. Design Parameters and Errors, Example B-2

	Design Parameters K_θ	K_Z	$K_{\dot\theta}$	$K_{\dot Z}$	λ_s^2	RMS % Difference (%) W	$\dot\theta$	θ	Z
Baseline	0.0	0.0	0.0	0.0	1.0	0.0	96.7	186.6	----
Priori	0.0	0.0	0.0	0.0	1.0	-	-	-	-
Optimal Design	4.2	.085	-4.12	-2.36	6.51	100	131.1	43.5	11.06

Table 23. Program Data, Example B-2

FACTOR	0.01	P	ADP1
NS	5	MAXER	200
IAOPT	1	MC	2
DELTA	0.5	NPT	200
MAXR	16	UNKR	5

The desired and final responses are given in figure 18.

Example B-3

In this example, feedback and geometry optimization is performed for

$$\hat{R} = [\hat{K}_{\dot Z},\hat{K}_{\dot\theta},\hat{K}_Z,\hat{K}_\theta,\hat{\lambda}_s^2]^T$$

with priori values for the parameters based on the results obtained in section 3C (pg. 16). The results are given in Table 24; the program settings are listed in Table 25.

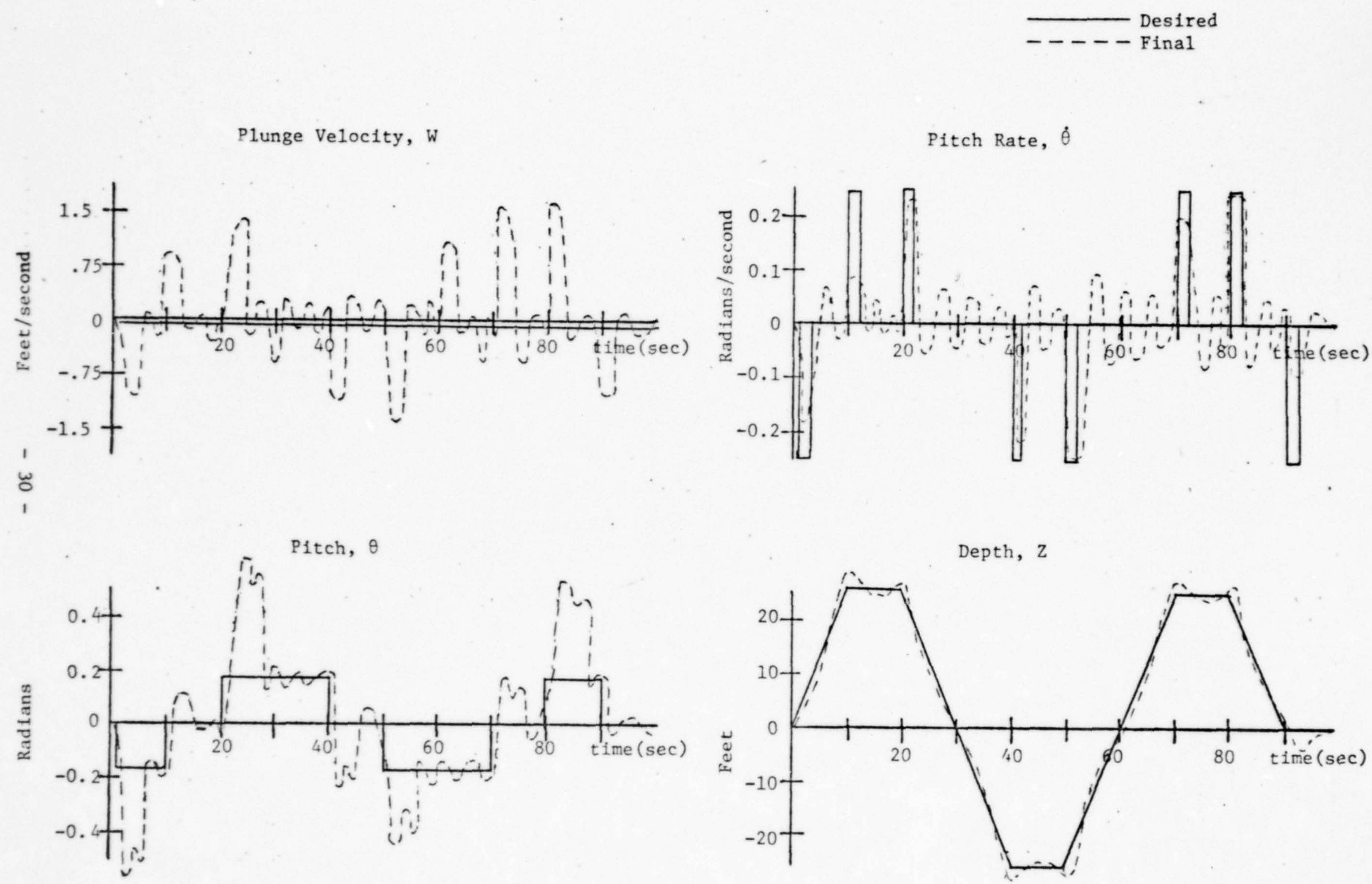


Figure 18. Comparison of Desired and Final Responses, Example B-2

Table 24. Design Parameters and Errors, Example B-3

	Design Parameters					RMS % Difference (%)			
	K_θ	K_Z	$K_{\dot\theta}$	$K_{\dot Z}$	λ_s^2	W	$\dot\theta$	θ	Z
Baseline	9.93	0.0	.115	0.5	2.03	0.0	96.7	186.6	----
Priori	9.93	0.0	.115	0.5	7.03	-	-	-	-
Optimal Design	5.48	1.106	.113	.224	9.88	100	105.4	46.3	6.82

Table 25. Program Data, Example B-3

FACTOR	0.05	P	ADP1
NS	5	MAXER	200
IAOPT	1	MC	2
DELTA	0.5	NPT	200
MAXR	16	UNKR	5

The desired and final responses are given in figure 19.

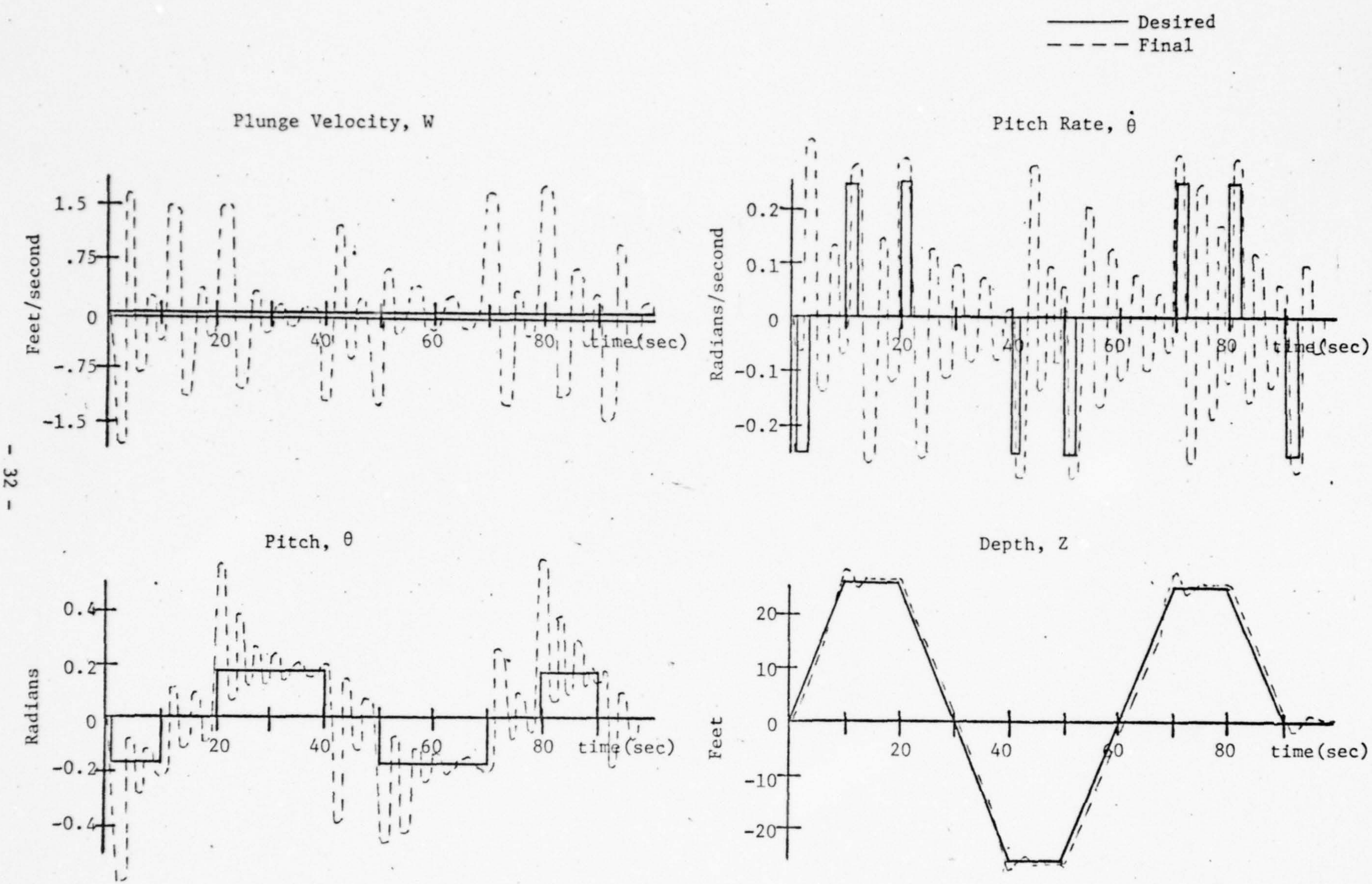


Figure 19. Comparison of Desired and Final Responses, Example B-3

V. PARAMETER OPTIMIZATION UTILIZING EXPONENTIAL TRANSFORMAION

In this phase of investigation, an exponential transformation is implemented to guarantee the acquisition of non-negative design parameters. Consider the transformation

$$\hat{p}_i = e^{r_i} = e^{\bar{r}_i \cdot \hat{r}_i}$$

$$\frac{\partial \hat{p}_i}{\partial \hat{r}_i} = \bar{r}_i \; e^{r_i} \tag{23}$$

where $\hat{p}$ represents the actual physical parameters and r represents the transformed variables. This transformation provides an isomorphic mapping from the real numbers (r) to the non-negative real numbers (p). Thus, if optimization is now performed on R^T, the optimal design parameters must necessarily be non-negative. It should be noted that each element in the H_2 matrix given in (10) can be calculated as follows

$$\frac{\partial c}{\partial \hat{r}} = \frac{\partial c}{\partial \hat{p}} \cdot \frac{\partial \hat{p}}{\partial \hat{r}} = \frac{\partial c}{\partial \hat{p}} \cdot \begin{bmatrix} r_i \; e^{r_i} & & \\ & \ddots & \\ & & \ddots \end{bmatrix} \tag{24}$$

A difficulty encountered in implementing this transformation is the selection of limits for allowable values for r and p. The necessity of limits arises from the following observations: (1) as p approaches zero, r approaches negative infinity, (2) if p takes on values close to 1.0, r becomes very small, (3) moderate values of r greater than 1.0 will result in large values of p.

The limits chosen to control the parameters are

$$\text{if } r < -18.4 \;,\; r = -18.4 \tag{25a}$$

$$\text{if } |r| < 10^{-10},\; r = 0.0 \tag{25b}$$

$$\text{if } r > 6.0 \;,\; r = 6.0 \tag{25c}$$

Equations (25a) and (25c) define the range of allowable r values. This corresponds to

$$p_{max} \simeq 403$$

$$p_{min} \simeq 10^{-7}$$

Equation (25b) restraints p from assuring values very close to 1.0, while still allowing it to be exactly 1.0.

Example 1

In this example, feedback and geometry optimization is performed for

$$\hat{P} = \left[K_{\dot{Z}} , K_{\dot{\theta}} , K_Z , K_\theta , \lambda_s^2 \right]^T$$

where the depth command (Z_{com}) and desired depth response is given in figure 16. The results are given in Table 26; the program settings are listed in Table 27.

Table 26. Design Parameters and Errors, Example 1

	Design Parameters K_θ	K_Z	$K_{\dot{\theta}}$	$K_{\dot{Z}}$	λ_s^2	RMS % Difference (%) W	$\dot{\theta}$	θ	Z
Baseline	0.01	0.01	0.01	0.01	1.0	0.0	20.99	99.43	98.83
Priori	0.01	0.01	0.01	0.01	1.0	-	-	-	-
Optimal Design	9.38	1.31	0.01	.0068	7.43	100.0	166.0	49.14	6.62

Table 27. Program Data, Example 1

FACTOR	0.01	P	ADP1
NS	5	MAXER	10
IAOPT	1	MC	2
DELTA	0.5	NPT	200
MAXR	16	UNKR	5

Notice the selection of Baseline and Priori parameters were chosen to avoid the problem areas controlled by the limits in (25). The one exception is the selection of 1.0 for λ_s^2 which yields and r value of exactly zero. The desired and final responses are given in figure 20.

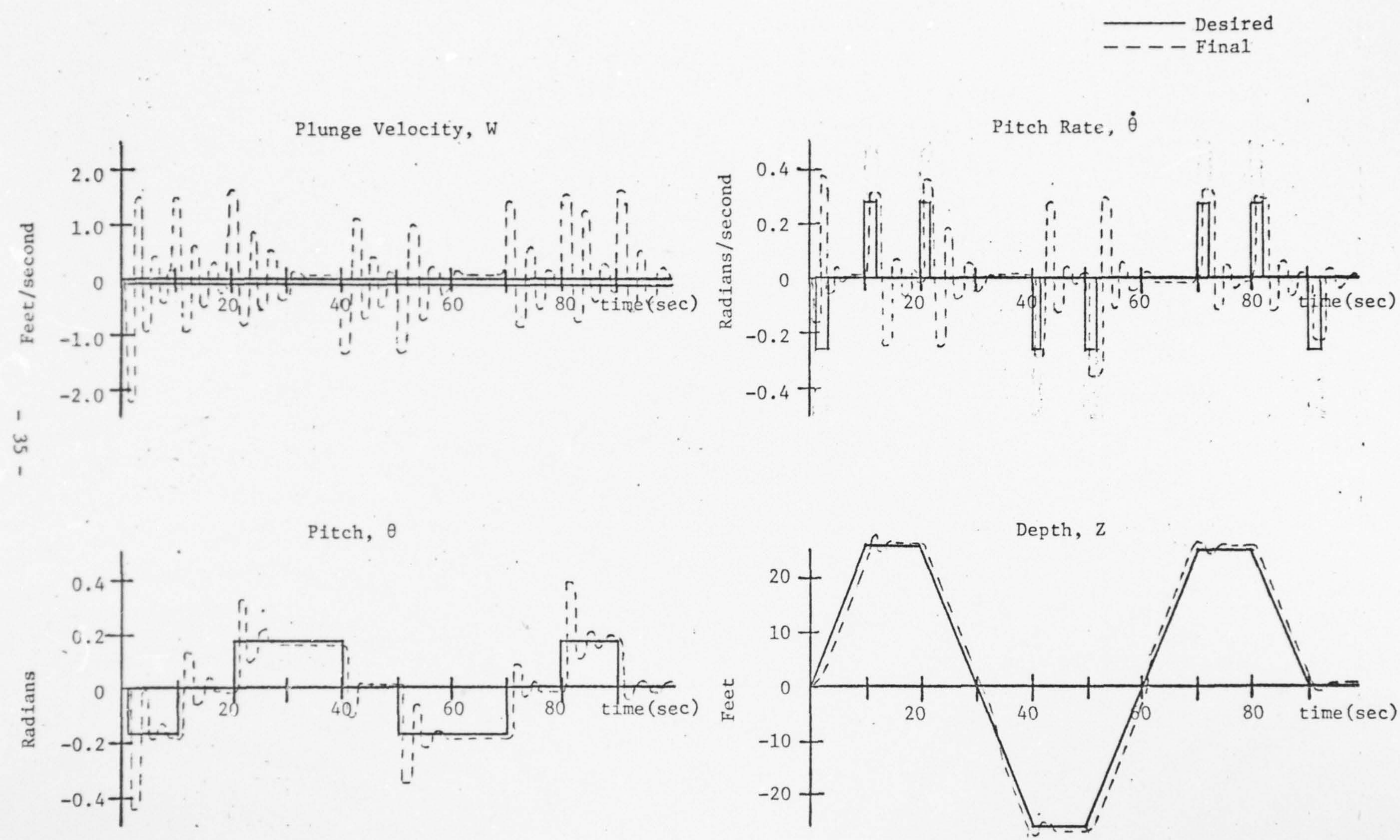


Figure 20. Compairson of Desired and Final Responses, Example 1

Example 2

This example is identical to the previous example with the exception that the prior guess for λ_s^2 is chosen to avoid 1.0. The results are given in Table 28, the program settings are listed in Table 29.

Table 28. Design Parameters and Errors, Example 2

	Design Parameters					RMS % Difference (%)			
	K_θ	K_Z	$K_{\dot\theta}$	$K_{\dot Z}$	λ_s^2	W	$\dot\theta$	θ	Z
Baseline	0.01	0.01	0.01	0.01	1.0	0.0	20.99	99.43	98.83
Priori	0.01	0.01	0.01	0.01	0.1	-	-	-	-
Optimal Design	16.36	2.82	.0091	.0072	17.63	100	158.9	40.92	5.02

Table 28. Program Data, Example 2

FACTOR	0.01	P	ADP1
NS	5	MAXER	10
IAOPT	1	MC	2
DELTA	0.5	NPT	200
MAXR	16	UNKR	5

Note that while the values of $K_{\dot\theta}$ and $K_{\dot Z}$ are practically the same as in the previous example, the values of K_θ, K_Z, and λ_s^2 increased significantly, resulting in a slightly better depth response. The desired and final responses are given in figure 21.

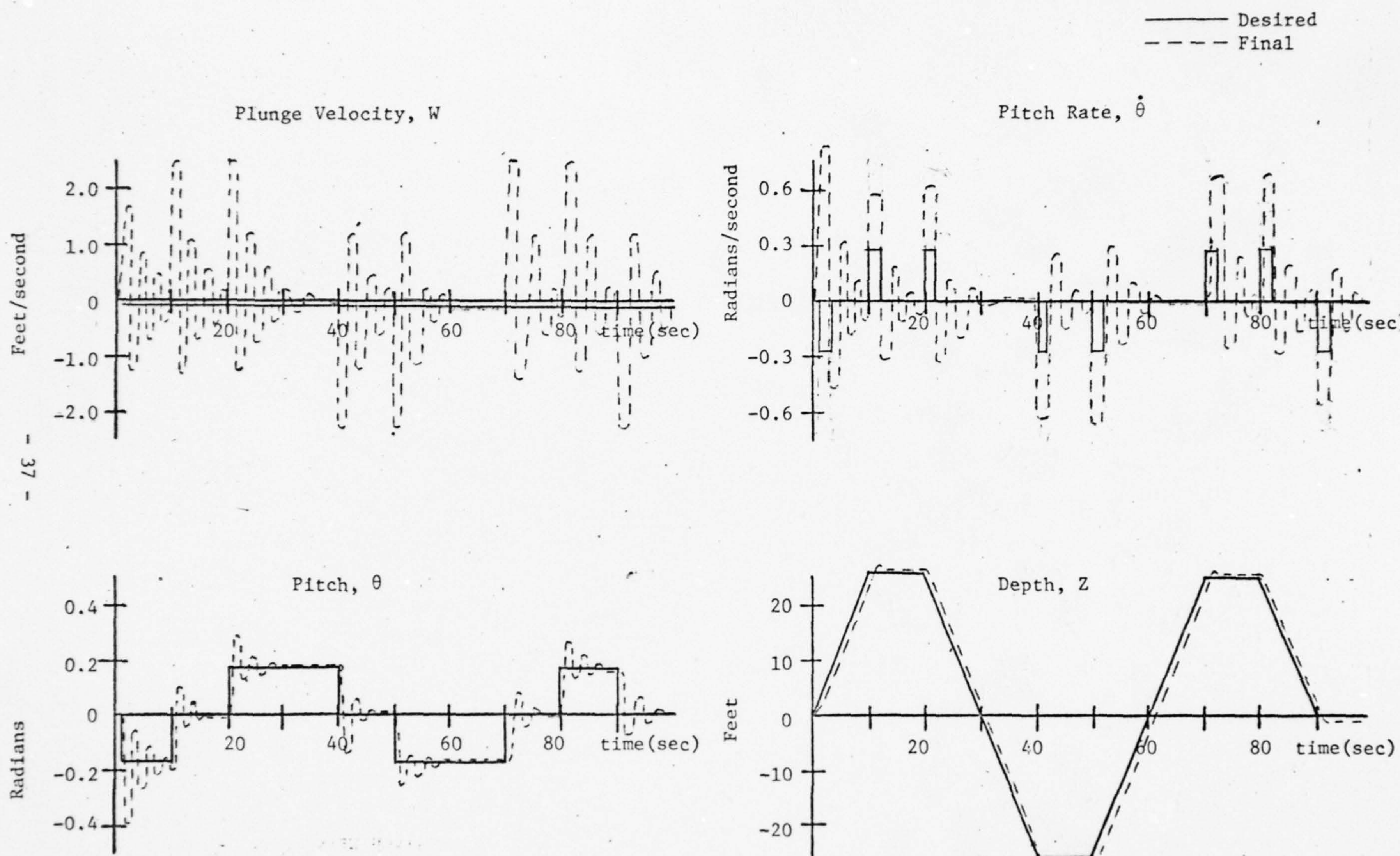


Figure 21. Comparison of Desired and Final Responses, Example 2

CONCLUSIONS

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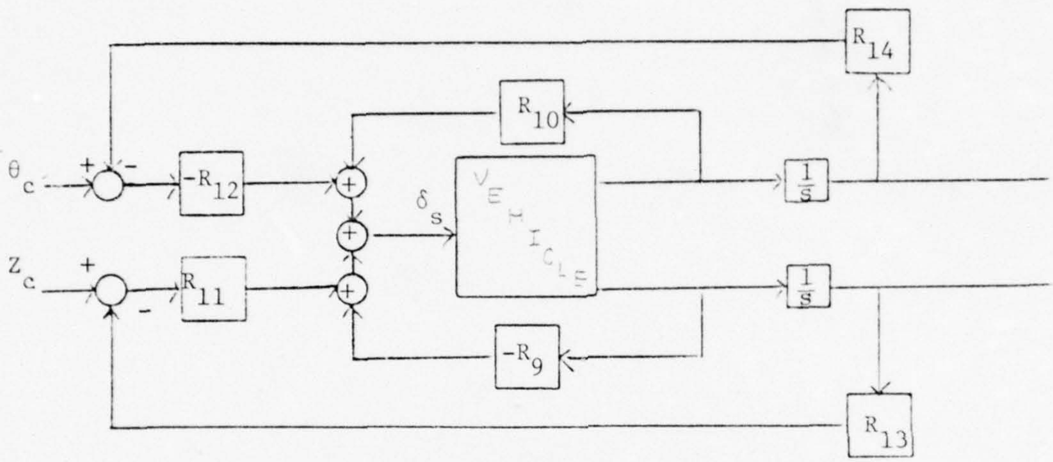
APPENDIX A

Description of Design Parameters

Geometry Parameters

Coning tower height	λ_c^2	- R_1
Bow-plane	λ_b^2	- R_2
Stern-plane	λ_s^2	- R_3
Rudder	λ_r^2	- R_4



Stern Plane

$R_9 = K^s_{\dot{z}}$

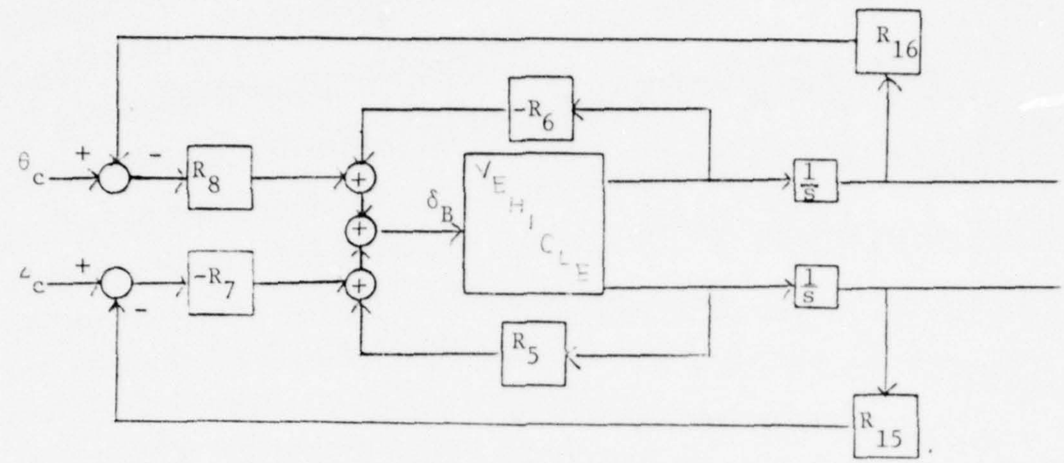
$R_{10} = K^s_{\dot{\theta}}$

$R_{11} = K^s_z$

$R_{12} = K^s_\theta$

$R_{13} = K_1$

$R_{14} = K_2$



Bow Plane

$R_5 = K^b_{\dot{z}}$

$R_6 = K^b_{\dot{\theta}}$

$R_7 = K^b_z$

$R_8 = K^b_\theta$

$R_{15} = K_3$

$R_{16} = K_4$

APPENDIX B

Iteration Data for MOF-NP

KOPT	NTER	JOPT1	JOPT2	INTR	IPROPT	IAOPT
1	2	0	1	1	1	1
0	2	0	2	1	1	1
0	2	0	4	1	1	1
0	2	0	8	1	1	1
0	2	0	16	1	1	1
0	2	0	32	1	1	1
0	2	0	64	1	1	1
0	2	0	128	1	1	1
0	2	0	500	1	1	1
0	2	1	0	1	1	1
0	2	1	0	2	1	1
0	2	1	0	4	1	1
0	2	1	0	5	1	1

APPENDIX C

The linearized state equations of a vehicle are of the form

$$A\frac{dx}{dt} = Bx + Cu \tag{C.1}$$

where

$$u = D \begin{matrix} \theta_{com} \\ Z_{com} \end{matrix} + Ex \tag{C.2}$$

Equation (C.1) can therefore be rewritten as

$$A\frac{dx}{dt} = (B+CE)x + CD \begin{matrix} \theta_{com} \\ Z_{com} \end{matrix} \tag{C.3}$$

The matricies of (C.3) are as follows:

$$A = \begin{bmatrix} m-X_{\dot{u}} & 0 & 0 & 0 & 0 \\ 0 & m-Z_{\dot{w}} & -Z_{\dot{q}} & 0 & 0 \\ 0 & -M_{\dot{w}} & I_y-M_{\dot{q}} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{C.4}$$

$$B = \begin{bmatrix} X_u & X_w & X_q & X_\theta & 0 \\ Z_u & Z_w & Z_q+mU_o & Z_\theta & 0 \\ M_u & M_w & M_q & M_\theta & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \tag{C.5}$$

$$C = \begin{bmatrix} X_{\delta_b} & X_{\delta_s} \\ Z_{\delta_b} & Z_{\delta_s} \\ M_{\delta_b} & M_{\delta_s} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{C.6}$$

$$D = \begin{bmatrix} K_\theta^b & -K_Z^b \\ -K_\theta^s & K_Z^s \end{bmatrix} \tag{C.7}$$

$$E = \begin{bmatrix} 0 & +K_{\dot{Z}}{}^b & K_{\dot{\theta}}{}^b & -K_\theta{}^b - K_{\dot{Z}}{}^b Uo & K_Z{}^b \\ 0 & -K_{\dot{Z}}{}^s & K_{\dot{\theta}}{}^s & K_\theta^s + K_{\dot{Z}}{}^s Uo & -K_Z{}^s \end{bmatrix} \tag{C.8}$$

Next, redefine matricies B and C as follows

$$\begin{aligned} B_{NEW} &= B+CE \\ C_{NEW} &= CD \end{aligned} \tag{C.9}$$

Thus (C.3) becomes

$$A\frac{dx}{dt} = B_{NEW}\, X + C_{NEW} \begin{bmatrix} \theta_{com} \\ Z_{com} \end{bmatrix} \tag{C.10}$$

Therefore we have

$$C_{NEW} = \begin{bmatrix} K_\theta{}^b X_{\delta_b} - K_\theta{}^s X_{\delta_s} & K_Z{}^s X_{\delta_s} - K_Z{}^b X_{\delta_b} \\ K_\theta{}^b Z_{\delta_b} - K_\theta{}^s Z_{\delta_s} & K_Z{}^s Z_{\delta_s} - K_Z{}^b Z_{\delta_b} \\ K_\theta{}^b M_{\delta_b} - K_\theta{}^s M_{\delta_s} & K_Z{}^s M_{\delta_s} - K_Z{}^b M_{\delta_b} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{C.11}$$

$$
B_{NEW} = \begin{bmatrix}
X_u & X_W + K_{\dot{Z}}^{b} X_{\delta_b} - K_{\dot{Z}}^{s} X_{\delta_s} & X_q + K_{\dot{\theta}}^{s} X_{\delta_s} - K_{\dot{\theta}}^{b} X_{\delta_b} & X_q - (K_\theta^{b} + K_{\dot{Z}}^{b} U_o) X_{\delta_b} + (K_\theta^{s} + K_{\dot{Z}}^{s} Uo) X_{\delta_s} & K_Z^{b} X_{\delta_b} - K_Z^{s} X_{\delta_s} \\
Z_u & Z_W + K_{\dot{Z}}^{b} Z_{\delta_b} - K_{\dot{Z}}^{s} Z_{\delta_s} & Z_q + mU_o + K_{\dot{\theta}}^{s} Z_{\delta_s} - K_{\dot{\theta}}^{b} Z_{\delta_b} & Z_\theta - (K_\theta^{b} + K_{\dot{Z}}^{b} U_o) Z_{\delta_b} + (K_\theta^{s} + K_{\dot{Z}}^{s} U_o) Z_{\delta_s} & K_Z^{b} Z_{\delta_b} - K_Z^{s} Z_{\delta_s} \\
M_u & M_W + K_{\dot{Z}}^{b} M_{\delta_b} - K_{\dot{Z}}^{s} M_{\delta_s} & M_q + K_{\dot{\theta}}^{s} M_{\delta_s} - K_{\dot{\theta}}^{b} M_{\delta_b} & M_\theta - (K_\theta^{b} + K_{\dot{Z}}^{b} U_o) M_{\delta_b} + (K_\theta^{s} + K_{\dot{Z}}^{s} U_o) M_{\delta_s} & K_Z^{b} M_{\delta_b} - K_Z^{s} M_{\delta_s} \\
0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & -U_o & 0
\end{bmatrix}
$$

APPENDIX D

The longitudinal dynamics of the USF-RPV vehicle are governed by the vector state equation

$$\begin{bmatrix} m-Z_{\dot{w}} & -Z_{\dot{q}} & 0.0 & 0.0 \\ -M_{\dot{w}} & I_y-M_{\dot{q}} & 0.0 & 0.0 \\ -0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} W \\ \dot{\theta} \\ \theta \\ Z \end{bmatrix} =$$

$$\begin{bmatrix} Z_w & Z_q+mU_o & Z_\theta & 0.0 \\ M_w & M_q & M_\theta & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 1.0 & 0.0 & -U_o & 0.0 \end{bmatrix} \begin{bmatrix} w \\ \dot{\theta} \\ \theta \\ Z \end{bmatrix} + \begin{bmatrix} Z\delta_b & Z\delta_s \\ M\delta_b & M\delta_s \\ 0.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix} \begin{bmatrix} \delta_b \\ \delta_s \end{bmatrix}$$

where

m = 280.39 slugs

I_y = 14267.0 slug ft^2

U_o = 14.616 ft/sec.

$Z_{\dot{w}} = -270.87369$

$Z_{\dot{q}} = M_{\dot{w}} = -49.718618$

$M_{\dot{q}} = -12818.43975$

$$Z_w = -[(5.693230U_o) + \lambda_c^2 (.037748\ U_o) + \lambda_B^2 (4.76436\ U_o) + \lambda_s^2 (6.01750\ U_o) + \lambda_R^2 (.030661\ U_o)]$$

$$M_w = (192.15600\ U_o) + \lambda_B^2 (23.095907\ U_o) + \lambda_s^2 (-85.5179\ U_o)$$

$$Z_q = -[(56.463697\ U_o) + \lambda_B^2 (-22.26927\ U_o) + \lambda_s^2 (111.627245\ U_o)]$$

$$M_q = (-931.456809\ U_o) + \lambda_B^2 (-108.451145\ U_o) + \lambda_s^2 (-1596.298837\ U_o)$$

$Z_\theta = 0.0$

$M_\theta = -722.2$

$Z_{\delta_b} = -\lambda_b^2 (5.591056)\ U_o^2$

$$Z_{\delta_s} = -\lambda_s^2 \; (2.503594) \; U_o^2$$

$$M_{\delta_b} = \lambda_b^2 \; (27.228202) \; U_o^2$$

$$M_{\delta_s} = \lambda_s^2 \; (-37.203137) \; U_o^2$$

APPENDIX E

***** CHANGES IN SUBROUTINE IDENT *****

```
          DO1ITER=1,NTER
          WRITE(6,403)ITER
  403     FORMAT(/1X,20(5HITER ),/30X,'ITERATION NO.',I5,
         1/30X,18(1H-),/30X,18(1H-))
          IF(ITRAN.EQ.0) GO TO 509
          DO 503 I=1,IUNKR
          J=INDXR(I)
          IF(R(J).GE.6.0) R(J)=6.0
          IF(R(J).LE.-18.4) R(J)=-18.4
          IF(RC(J).GE.6.0) RC(J)=6.0
          IF(RC(J).LE.-18.4) RC(J)=-18.4
          IF(ABS(R(J)).LT.1.E-10) R(J)=0.0
  508     CONTINUE
  503     CONTINUE
          CALL SAVER
          ILOG2=0
          IPOS=1
          CALL SELECT
          CALL ERROR(YY,SUMER,ICH)
C
C             CALCULATE THE WEIGHTING MATRIX Q USED IN J1
C
          DO105I=1,NTM
          TEM=QQ(I)
          IF(TEM.EQ.0.)TEM=1..
          Q(I)=1./(FLOAT(NTM)*TEM)
          Q(1)=0.0
          Q(2)=0.0
          Q(3)=0.0
          Q(4)=0.0
C         Q(5)=0.0
          WRITE(6,1500)Q(I)
  105     CONTINUE
  1500    FORMAT(10X,'Q EQUALS ',G14.6)
C
C
C             CALCULATE THE WEIGHTING MATRIX Q2 USED IN J2
C
          PARER=0.0
          ZARER=0.0
          WARER=0.0
          DO805IPMC=1,IUNKR
          J=INDXR(IPMC)
          TEM=(PRIORE(IPMC)-R(J)*RC(J))**2
          TEM1=(PRIORE(IPMC))**2
          IF(TEM1.NE.0.0)GO TO 965
          TEM1=1.0
          ZARER=ZARER+TEM/TEM1
  965     CONTINUE
          Q2(IPMC)=1.0/TEM1
  805     PARER=PARER+TEM/TEM1
          WARER=PARER-ZARER
          IF(IADPT.EQ.0) GO TO 962
C
C               ADAPTIVE METHOD
C
          RARER=PARER
          FACTOR=TEM8
C
C          PMAX IS PROGRAM VARIABLE FOR MAXER
C
          PMAX=200.0
          IF(ITRAN.EQ.1) PMAX=10.0
          IF(RARER.LE.0.2)FACTOR=0.0
          IF(RARER.GE.PMAX)FACTOR=1.0
          IF(TEM8.LE.0.00001)FACTOR=TEM8
```

```
962   CONTINUE
      IF(IPRNT.AGE.1)WRITE(6,930)PARER
      IF(IPRNT.GE.1)WRITE(6,966)ZARER
      IF(IPRNT.GE.1)WRITE(6,967)WARER
930   FORMAT(17X,'PARER = ',G14.6)
966   FORMAT(17X,'PARER1=',G14.6)
967   FORMAT(17X,'PARER2=',G14.6)
      IF(PARER.EQ.0.0) PARER=FLOAT(IUNKR)
      DO806IPMC=1,IUNKR
      IF(IADPT.LE.1) GO TO 963
C
C          JOPT2 ADJUSTMENT
C
      IF(JOPT1.EQ.1)GO TO 960
      IF(JOPT2.LE.8)FACTOR=0.0
960   CONTINUE
963   CONTINUE
C
      Q2(IPMC)=Q2(IPMC)*FACTOR/PARER
      WRITE(6,1501)IPMC,Q2(IPMC)
806   CONTINUE
1501  FORMAT(20X,I2,' Q2 EQUALS ',G14.6)
      DO4I=1,NTP1
      DO4J=1,NTP1
4     G(I,J)=0.
      DO5I=1,NT
      X(I)=0.
5     XD(I)=0.
      DO6I=1,NA
```

```
      SUBROUTINE PDERCR
C     IMPLICIT REAL*8(A-H,O-Z)
      COMMON/WORK4/IUNKR,IPOS,ITRAN
      COMMON/WORK5/INDXR(10)
      COMMON/WORK3/R(16),RC(16),H(20,20),RS(16),HT(20,20),GN(20,20),
     1P(16),PC(16),PS(16)
      COMMON /MATRIX/A(10,10),AI(10,10),B(10,10),C(10,15),
     1AS(10,10),BS(10,10),B1(10,10),B2(10,10),CS(10,15),C1(10,15),
     2C2(10,15),CA(10,10),CB(10,10),CC(10,15),XINT(10),SB(10),SDB(10),
     3BINP(10),XINTS(10),SBS(10),SDBS(10),BINPS(10),Q(20),QQ(20),Q1(20)
      COMMON /INTGS/MAX,NA,MC,NPUTS,IPARM(100,5),NPAB,NPABC,NTER,INTR,
     1JOPT1,JOPT2,IAOPT,MAXG,IS(10),ISD(10),ISM(10),NS,ISDM(10),NSD,
     2INTS(10),NI,ISB(10),NSB,ISDB(10),NSDB,INPB(10),NINB,NPT,MAXNPT,
     3NTP,NTP1,NT,NTM,NS1,NS2,NS3,NS4,IPRNT,ILOG1,ILOG2,IADPT
       DIMENSION IND(10),H3(20,20)
C
C
C     THIS SUBROUTINE CALCULATES THE PARTIAL DERIVATIVE OF C
C     (THE VECTOR OF UNKNOWN PARAMETERS) WITH RESPECT TO R
C
C
C
      DO 945 II=1,IUNKR
      I=INDXR(II)
      RRC=R(I)*RC(I)
      IF(RRC.GE.6.0) RRC=6.0
      P(I)=EXP(RRC)
      PC(I)=1.0
      IF(ITRAN.EQ.0) P(I)=R(I)
      IF(ITRAN.EQ.0) PC(I)=RC(I)
  945 CONTINUE
      WRITE(6,946)
      CALL PRVEC(P,16)
      WRITE(6,947)
  946 FORMAT(/10X,'P VECTOR',/)
  947 FORMAT(/10X,'R VECTOR',/)
      CALL PRVEC(R,16)
      UCOM=14.816
      UCOM2=UCOM**2
C
C
C     FORM  X DELTAB ETC.
C
      XDB=-P(2)*PC(2)*.559106*UCOM2
      ZDB=-P(2)*PC(2)*5.59106*UCOM2
      MDB=P(2)*PC(2)*27.228202*UCOM2
C
C
      XDS=-P(3)*PC(3)*.250359*UCOM2
      ZDS=-P(3)*PC(3)*2.503594*UCOM2
      MDS=P(3)*PC(3)*(-37.203137)*UCOM2
C
      TT=-P(13)*PC(13)*P(11)*PC(11)
      SS=P(14)*PC(14)*P(12)*PC(12)+P(9)*PC(9)*UCOM
C
      DO 1I=1,20
      DO1J=1,NPABC
      H(I,J)=0.0
      H3(I,J)=0.0
    1 CONTINUE
C
C
      IND(1)=1
      IND(2)=2
      IND(3)=3
      IND(4)=4
```

```
      IND(5)=5
      H(1,1)=-PC(9)*XDS
      H(1,3)= PC(9)*UCOM*XDS
      H(1,5)=-PC(9)*ZDS
      H(1,7)= PC(9)*UCOM*ZDS
      H(1,9)=-PC(9)*MDS
      H(1,11)= PC(9)*UCOM*MDS
C
C
      H(2,2)= PC(10)*XDS
      H(2,6)= PC(10)*ZDS
      H(2,10)= PC(10)*MDS
C
C
C
      H(3,4)=-PC(11)*XDS*P(13)*PC(13)
      H(3,8)=-PC(11)*ZDS*P(13)*PC(13)
      H(3,12)=-PC(11)*MDS*P(13)*PC(13)
C
C
      H(3,14)= PC(11)*XDS
      H(3,16)= PC(11)*ZDS
      H(3,18)= PC(11)*MDS
C
C
      H(4,3)= PC(12)*XDS*P(14)*PC(14)
      H(4,7)= PC(12)*ZDS*P(14)*PC(14)
      H(4,11)= PC(12)*MDS*P(14)*PC(14)
      H(4,13)=-PC(12)*XDS
      H(4,15)=-PC(12)*ZDS
      H(4,17)=-PC(12)*MDS
C
C
      IF(ABS(P(3)).LE.0.00001)GO TO 918
      H(5,1)=-P(9)*PC(9)*XDS/P(3)
      H(5,2)=-0.01*PC(3)*(111.627245*UCOM)+P(10)*PC(10)*XDS/P(3)
      H(5,3)=SS*XDS/P(3)
      H(5,4)=TT*XDS/P(3)
      H(5,5)=-1.0*PC(3)*(6.01750*UCOM)-P(9)*PC(9)*ZDS/P(3)
      H(5,6)=-PC(3)*(111.627245*UCOM)+P(10)*PC(10)*ZDS/P(3)
      H(5,7)=SS*ZDS/P(3)
      H(5,8)=TT*ZDS/P(3)
      H(5,9)=PC(3)*(-85.5179*UCOM)-P(9)*PC(9)*MDS/P(3)
      H(5,10)=PC(3)*(-1596.298837*UCOM)+P(10)*PC(10)*MDS/P(3)
      H(5,11)=SS*MDS/P(3)
      H(5,12)=TT*MDS/P(3)
C
C
      H(5,13)=-P(12)*PC(12)*XDS/P(3)
      H(5,14)=P(11)*PC(11)*XDS/P(3)
      H(5,15)=-P(12)*PC(12)*ZDS/P(3)
      H(5,16)=P(11)*PC(11)*ZDS/P(3)
      H(5,17)=-P(12)*PC(12)*MDS/P(3)
      H(5,18)=P(11)*PC(11)*MDS/P(3)
C
C
      GO TO 919
  918 CONTINUE
      WRITE(6,920)
  920 FORMAT(/10X,'************ERROR IN PDERCR-P(3)=0***********',/)
  919 CONTINUE
      DO 925 I=1,IUNKR
      DO 925 J=1,NPABC
      II=IND(I)
      H3(I,J)=H(II,J)
  925 CONTINUE
```

```
      DO 926 I=1,5
      DO 926 J=1,NPABC
      H(I,J)=0.0
  926 CONTINUE
      DO 927 I=1,IUNKR
      DO 927 J=1,NPABC
      H(I,J)=H3(I,J)
  927 CONTINUE
      WRITE(6,941)
  941 FORMAT(//20X,'THE H MATRIX AFTER H3 ADJUSTMENT',/)
      DO 942 I=1,IUNKR
      WRITE(6,943)(H(I,J),J=1,NPABC)
  942 CONTINUE
  943 FORMAT(1X,8G14.6)
      DO 944 I=1,IUNKR
      II=INDXR(I)
      PRC=P(II)*PC(II)*RC(II)
      IF(ITRAN.EQ.0) PRC=1.0
      DO 944 J=1,NPABC
      H(I,J)=H(I,J)*PRC
  944 CONTINUE
      WRITE(6,948)
      DO 949 I=1,IUNKR
      WRITE(6,950) (H(I,J),J=1,NPABC)
  949 CONTINUE
  948 FORMAT(/20X,'THE H3 MATRIX AFTER TRANSFORMATION',/)
  950 FORMAT(1X,8G14.6)
      RETURN
      END
```

```
      SUBROUTINE DESIGN
C     IMPLICIT REAL*8(A-H,O-Z)
      COMMON/WORK3/R(16),RC(16),H(20,20),RS(16),HT(20,20),GN(20,20),
     1P(16),PC(16),PS(16)
      COMMON /MATRIX/A(10,10),AI(10,10),B(10,10),C(10,15),
     1AS(10,10),BS(10,10),B1(10,10),B2(10,10),CS(10,15),C1(10,15),
     2C2(10,15),CA(10,10),CB(10,10),CC(10,15),XINT(10),SB(10),SDB(10),
     3BINP(10),XINTS(10),SBS(10),SDBS(10),BINPS(10),Q(20),QQ(20),Q1(20)
      COMMON /INTGS/MAX,NA,MC,NPUTS,IPARM(100,5),NPAB,NPABC,NTER,INTR,
     1JOPT1,JOPT2,IAOPT,MAXG,IS(10),ISD(10),ISM(10),NS,ISDM(10),NSD,
     2INTS(10),NI,ISB(10),NSB,ISDB(10),NSDB,INPB(10),NINB,NPT,MAXNPT,
     3NTP,NTP1,NT,NTM,NS1,NS2,NS3,NS4,IPRNT,ILOG1,ILOG2,IADPT
      COMMON/KFDB/E(5,5),CE(5,5),D(5,5)
      COMMON/WORK4/IUNKR,IPOS,ITRAN
      COMMON/WORK5/INDXR(10)
C
C
C     THIS SUBROUTINE FORMS THE B AND C MATRICES BASED UPON THE DESIGN
C
C
C
      IF(IPOS.EQ.0) GO TO 13
      IF(IPOS.EQ.2) GO TO 9
      DO 8 II=1,IUNKR
      I=INDXR(II)
      RRC=R(I)*RC(I)
      IF(RRC.GE.6.0) RRC=6.0
      P(I)=EXP(RRC)
      PC(I)=1.0
      IF(ITRAN.EQ.0) P(I)=R(I)
      IF(ITRAN.EQ.0) PC(I)=RC(I)
    8 CONTINUE
      GO TO 13
    9 DO 12 II=1,IUNKR
      I=INDXR(II)
      IF(P(I).LE.1.E-8) GO TO 10
      IF(RC(I).LE.1.E-8) RC(I)=1.0
      R(I)=ALOG(P(I))/RC(I)
      GO TO 11
   10 R(I)= -18.4
   11 CONTINUE
      IF(ITRAN.EQ.0) R(I)=P(I)
      IF(ITRAN.EQ.0) RC(I)=PC(I)
   12 CONTINUE
   13 CONTINUE
      UCOM=14.616
      DO1I=1,NA
      DO1J=1,NA
      B(I,J)=0.0
    1 CONTINUE
      DO2I=1,NA
      DO2J=1,MC
      C(I,J)=0.0
    2 CONTINUE
      ZQ=-1.0*((56.463697*UCOM)+P(2)*PC(2)*(-22.269270*UCOM)+
     1P(3)*PC(3)*(111.627245*UCOM))
      B(1,1)=-1.0*((1.182744*UCOM)+P(1)*PC(1)*(0.075873*UCOM)+
     1P(2)*PC(2)*(0.044108*UCOM)+P(3)*PC(3)*(0.074758*UCOM)+P(4)*PC(4)*
     2(0.061628*UCOM))
      B(1,1)=-21.0340
      B(1,3)=0.01*ZQ
      B(1,4)=5.8721
      B(2,2)=-1.0*((5.69323*UCOM)+P(1)*PC(1)*(0.037748*UCOM)+P(2)*PC(2)
     1*(4.764360*UCOM)+P(3)*PC(3)*(6.01750*UCOM)+P(4)*PC(4)*
     2(0.0306606*UCOM))
      B(2,3)=ZQ+280.39*UCOM
      B(3,2)=(192.156*UCOM)+P(2)*PC(2)*(23.095900*UCOM)+P(3)*PC(3)*
```

```
     1(-35.5179*UCOM)
      B(3,3)=-931.456809*UCOM+P(2)*PC(2)*(-108.451145*UCOM)+
     1P(3)*PC(3)*(-1596.298837*UCOM)
      B(3,4)=-722.2
      B(4,3)=1.0
      B(5,2)=1.0
      B(5,4)=-UCOM
      C(1,1)=-P(2)*PC(2)*(0.559106)*(UCOM**2)
      C(1,2)=-P(3)*PC(3)*(0.250359)*(UCOM**2)
      C(2,1)=-P(2)*PC(2)*(5.591056)*(UCOM**2)
      C(2,2)=-P(3)*PC(3)*(2.503594)*(UCOM**2)
      C(3,1)=P(2)*PC(2)*(27.228202)*(UCOM**2)
      C(3,2)=P(3)*PC(3)*(-37.203137)*(UCOM**2)
C
C     FORM   E MATRIX
C
      E(1,1)=0.0
      E(2,1)=0.0
      E(1,2)=P(5)*PC(5)
      E(1,3)=-P(6)*PC(6)
      E(1,4)=-P(16)*PC(16)*P(8)*PC(8)-P(5)*PC(5)*UCOM
      E(1,5)=P(15)*PC(15)*P(7)*PC(7)
      E(2,2)=-P(9)*PC(9)
      E(2,3)=P(10)*PC(10)
      E(2,4)=P(14)*PC(14)*P(12)*PC(12)+P(9)*PC(9)*UCOM
      E(2,5)=-P(13)*PC(13)*P(11)*PC(11)
C
C     MULTIPLY C AND E MATRIX
C
      DO 3I=1,NA
      DO3J=1,NA
      CE(I,J)=0.0
      DO 3K=1,MC
      CE(I,J)=CE(I,J) + C(I,K)*E(K,J)
    3 CONTINUE
C
C     FORM OVERALL B MATRIX
C
      DO 4I=1,NA
      DO 4J=1,NA
      B(I,J)=B(I,J)+CE(I,J)
    4 CONTINUE
C
C     FORM D MATRIX
C
      D(1,1)=P(8)*PC(8)
      D(1,2)=-P(7)*PC(7)
      D(2,1)=-P(12)*PC(12)
      D(2,2)=P(11)*PC(11)
C
C     FORM NEW CONTROL MATRIX
C
      DO 5I=1,NA
      DO 5J=1,MC
      CE(I,J)=0.0
      DO 5K=1,MC
      CE(I,J)=CE(I,J) + C(I,K)*D(K,J)
    5 CONTINUE
C
C     FORM OVERALL C MATRIX
C
      DO 6I=1,NA
      DO 6J=1,MC
      C(I,J)=CE(I,J)
    6 CONTINUE
C
      RETURN
      END
```